If the distance of the observer/listener from both of the two sound sources is *large* compared to the sound source separation distance, *i.e.* L >> d, {the so-called "far-field" limit} then tan  $\theta \approx \sin \theta$  and hence  $\Delta L \approx d \sin \theta$ . The relative phase difference between the two amplitudes at the observer/listener location is:  $\delta = k \Delta L = 2\pi \Delta L/\lambda \approx 2\pi d \sin \theta/\lambda$  (in radians).

At the observer/listener location, suppose the individual over-pressure amplitudes at a given instant in time are given by:

$$p_{1}(L,t) = p_{o_{1}} \cos(\omega t - kL)$$

$$p_{2}(L,t) = p_{o_{2}} \cos[\omega t - k(L + \Delta L)] = p_{o_{2}} \cos(\omega t - kL - \delta)$$

$$p_{tot}(L,t) = p_{1}(L,t) + p_{2}(L,t) = p_{o_{1}} \cos(\omega t - kL) + p_{o_{2}} \cos(\omega t - kL - \delta)$$

The phase-sensitive interference relation between the two individual over-pressure amplitudes and the resultant/total over-pressure amplitude heard by the observer/listener can be represented graphically using a so-called *phasor diagram*, as shown in the figure below. The phasor diagram adds the two vector amplitudes together to form the resultant/overall/net vector amplitude.

The phasor diagram, by convention, orients the over-pressure amplitude associated with the first sound source  $p_1(z,t)$  on the horizontal axis. The base of the over-pressure amplitude associated with the second sound source,  $p_2(z,t)$  is placed at the tip of the first, and angled away from the *x*-axis by the relative phase difference angle,  $\delta$ . The resultant/total/net displacement amplitude,  $y_{tot}(t)$  is the vector drawn from the base of the first displacement amplitude to the tip of the second displacement amplitude, as shown in the figure below:



Note that the phasor triangle obeys the trigonometrical *law of cosines* relation:

$$c^{2} = a^{2} + b^{2} - 2ab\cos(\pi - \delta) = a^{2} + b^{2} + 2ab\cos\delta$$

{the latter relation on the RHS of this equation was obtained using the trigonometric identity: cos(A - B) = cosA cosB + sinA sinB).

The *magnitude* (*i.e.* length) of the resultant/total/net over-pressure amplitude, *p*tot is given by:

$$p_{tot}^2 = p_{o_1}^2 + p_{o_2}^2 + 2p_{o_1}p_{o_2}\cos\delta$$
 or:  $p_{tot} = \sqrt{p_{o_1}^2 + p_{o_2}^2 + 2p_{o_1}p_{o_2}\cos\delta}$ 

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