

There are four possible/different/distinct cases/situations for the Doppler effect:

- a.) Sound source and observer are both moving in opposite directions – but approaching each other. The relative motion of sound source and observer is toward each other:

$$\begin{array}{ccc}
 \mathbf{U}_{\text{source}} & \mathbf{U}_{\text{observer}} & \\
 \longrightarrow & \longleftarrow & \\
 \end{array}
 \quad
 f_{\text{observer}} = \left( \frac{V_{\text{medium}} + U_{\text{observer}}}{V_{\text{medium}} - U_{\text{source}}} \right) f_{\text{source}}$$

- b.) Sound source and observer are both moving in opposite directions – but receding from each other. The relative motion of sound source and observer is away from each other:

$$\begin{array}{ccc}
 \mathbf{U}_{\text{source}} & \mathbf{U}_{\text{observer}} & \\
 \longleftarrow & \longrightarrow & \\
 \end{array}
 \quad
 f_{\text{observer}} = \left( \frac{V_{\text{medium}} - U_{\text{observer}}}{V_{\text{medium}} + U_{\text{source}}} \right) f_{\text{source}}$$

- c.) Sound source and observer are both moving in same direction, but the source is ahead of the observer:

$$\begin{array}{ccc}
 \mathbf{U}_{\text{observer}} & \mathbf{U}_{\text{source}} & \\
 \longrightarrow & \longrightarrow & \\
 \end{array}
 \quad
 f_{\text{observer}} = \left( \frac{V_{\text{medium}} + U_{\text{observer}}}{V_{\text{medium}} + U_{\text{source}}} \right) f_{\text{source}}$$

- d.) Sound source and observer both moving in same direction, but the source is behind the observer:

$$\begin{array}{ccc}
 \mathbf{U}_{\text{source}} & \mathbf{U}_{\text{observer}} & \\
 \longrightarrow & \longrightarrow & \\
 \end{array}
 \quad
 f_{\text{observer}} = \left( \frac{V_{\text{medium}} - U_{\text{observer}}}{V_{\text{medium}} - U_{\text{source}}} \right) f_{\text{source}}$$

A frequency shift  $\Delta f = f_{\text{observer}} - f_{\text{source}}$  occurs when the sound source and/or observer are in motion with respect to ground reference frame!

The frequency heard/perceived by observer is higher if the relative motion of the sound source and observer is toward each other:  $f_{\text{observer}} > f_{\text{source}}$ , thus  $\Delta f = f_{\text{observer}} - f_{\text{source}} > 0$ .

The frequency heard/perceived by observer is lower if the relative motion of the sound source and observer is away from each other:  $f_{\text{observer}} < f_{\text{source}}$ , thus  $\Delta f = f_{\text{observer}} - f_{\text{source}} < 0$ .

For each of above four cases, can get limiting/special cases, e.g. when ground speed of observer,  $U_{\text{observer}} = 0$  and/or when ground speed of sound source,  $U_{\text{source}} = 0$ .

If there exists a wind, then the component of wind velocity vector projected onto the line of relative motion between sound source and observer must be added (or subtracted) from ground speed of propagation of sound,  $V_{\text{sound}}$ . The presence/existence of wind has no effect if it is transverse (i.e. perpendicular) to the line defined by the relative motion between the sound source and observer.

Formally, the Doppler effect is actually a 3-D vector problem – involving the 3-D velocity vectors of all three items – i.e. the 3-D velocity vectors associated with the sound source, observer and the wind (if present). The above four 1-D formulae are correct only for the projections of these velocity vectors onto the line defined by the relative 1-D motion between sound source and observer.