a.) DC (*i.e*. zero frequency, $f = 0$ *Hz*) components (*i.e*. constant terms) present, associated with/arising from both of the individual amplitudes A_{10} and A_{20}.
b.) 2nd harmonic components present with $2f_1$ and $2f_2$, as well as:
c.) a component associated with the <u>*sum*</u> of the two frequencies, $\Omega_{21} = f_1 + f_2$, and:
d.) a component associated with the <u>*difference*</u> of the two frequencies, $\Delta f_{21} = f_1 - f_1$.
This is a remarkably similar result to that associated *e.g*. with the output response from a system having a <u>*quadratic*</u> <u>*non-linear*</u> <u>*response*</u> to a pure/single-frequency sine-wave input!
(Please see/read the Physics 406 Lecture Notes on Distortion for more details...)

So, simply stated, beats is a phenomenon where *e.g*. two waves from separate sound sources with slightly different frequencies are combined/allowed to *mix*. The resultant total/overall wave exhibits <u>*interference*</u> between the two waves of slightly different frequencies. We hear this interference effect as an amplitude modulation of the overall envelope of the waveform.

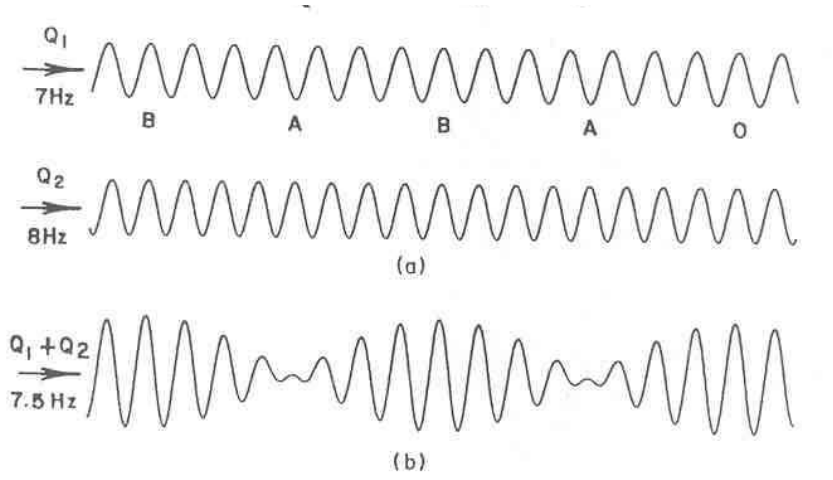

FIG. 20. Production of beats. (a) Waves from two separate sources of slightly different frequency. (b) Resultant wave formed by the superposition of the two separate waves.

We, as human beings hear/perceive the two frequencies as an <u>*average*</u> frequency, $< f_{avg} > = \frac{1}{2}(f_1+f_2)$, if the two signals have equal amplitude/intensities, and we hear/perceive the difference frequency as a beat frequency – an amplitude modulation effect of the overall waveform, with $\Delta f = f_{beat} = |f_1 - f_2|$ (= absolute value of the frequency difference). Fundamentally, beats is manifestly an <u>*interference phenomenon*</u> associated with two (or more) waves of <u>*nearly*</u> the same, but not identical frequency.

Note that the phenomenon of beats is <u>*not*</u> associated solely with acoustical physics – for example, in two <u>*entangled*</u> beams of light/EM waves of nearly the same frequency will also exhibit the property known as <u>*optical beats*</u> – this is the principal of operation of *LIDAR* (<u>*LI*</u>ght <u>*D*</u>etection <u>*A*</u>nd <u>*R*</u>anging), as well as Doppler *RADAR* (*RA*dio *D*ectection *A*nd *R*anging). In LIDAR, a laser beam is split into two separate beams, e.g. using a beam splitter – a <u>*reference*</u> beam and a <u>*probe*</u> beam – the latter of which reflects off of a <u>moving</u> object, resulting in a Doppler-shifted frequency (see below), which upon <u>mixing</u> with the reference beam, results in optical beats. Thus, e.g. police use *LIDAR* devices for (very accurately) monitoring the speed of vehicles on interstate highways…