one full revolution) behind in phase relative to phasor 1 (which has precessed through N_1 = 50.0 full revolutions), thus, the net/overall result is the same as being exactly in phase with phasor 1! At this point in time, $A_{tot}(t = \tau_{beat}) = A_1(t = \tau_{beat}) + A_2(t = \tau_{beat}) = 2A_1(t = \tau_{beat}) = 2A_1(t = \tau_{beat})$, and the phasor diagram looks precisely like that at time $t = 0$.

Thus, it should (hopefully) now be clear to the reader that the phenomenon of beats is manifestly that of time-dependent alternating constructive/destructive interference between two periodic signals of comparable frequency, at the amplitude level. This is by no means a trivial point, as often the beats phenomenon is discussed in physics textbooks in the context of intensity, $I_{tot}(t) = |A_{tot}(t)|^2 = |A_1(t) + A_2(t)|^2$. From the above discussion, the ***physics origin*** of the beats phenomenon has absolutely *nothing* to do with *intensity* of the overall/ resultant signal.

The primary reason that the phenomenon of beats is discussed more often in terms of intensity, rather than amplitude is that the physics is perhaps easier to understand from the intensity perspective – at least mathematically, things appear more obvious, physically:

$$I_{tot}(t) = \left|A_{tot}(t)\right|^2 = \left|A_1(t) + A_2(t)\right|^2$$
$$I_{tot}(t) = A_{10}^2\cos^2\left(\omega_1(t)t\right) + A_{20}^2\cos^2\left(\omega_2(t)t + \Delta\phi_{21}(t)\right) + 2A_{10}A_{20}\cos\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t + \Delta\phi_{21}(t)\right)$$

Let us define: $\vartheta_1(t) \equiv \omega_1(t)t$ and: $\vartheta_2(t) \equiv \left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)$

And then let us use the mathematical identity:

$$\cos\vartheta_1\cos\vartheta_2 \equiv \tfrac{1}{2}\left[\cos\left(\vartheta_2 + \vartheta_1\right) + \cos\left(\vartheta_2 - \vartheta_1\right)\right]$$

Thus:

$$\begin{aligned} I_{tot}(t) &= A_{10}^2\cos^2\left(\omega_1(t)t\right) + A_{20}^2\cos^2\left(\omega_2(t)t + \Delta\phi_{21}(t)\right) \\ &+ A_{10}A_{20}\left[\cos\left((\omega_2(t) + \omega_1(t))t + \Delta\phi_{21}(t)\right) + \cos\left((\omega_2 - \omega_1)t - \Delta\phi_{21}(t)\right)\right] \end{aligned}$$

The let us define:

$$\Omega_{21}(t) \equiv \left(\omega_2(t) + \omega_1(t)\right) \qquad \Delta\omega_{21}(t) \equiv \left|\omega_2(t) - \omega_1(t)\right|$$

We then obtain:

$$\boxed{I_{tot}(t) = A_{10}^2\cos^2\left(\omega_1(t)t\right) + A_{20}^2\cos^2\left(\omega_2(t)t + \Delta\phi_{21}(t)\right) + A_{10}A_{20}\left[\cos\left(\Omega_{21}(t)t + \Delta\phi_{21}(t)\right) + \cos\left(\Delta\omega_{21}(t)t - \Delta\phi_{21}(t)\right)\right]}$$

Using the above mathematical identity again, we see that:

$$\cos^2\vartheta = \cos\vartheta\cos\vartheta \equiv \tfrac{1}{2}\left[\cos 0 + \cos 2\vartheta\right] = \tfrac{1}{2}\left[1 + \cos 2\vartheta\right]$$

and thus we obtain an additional relation, one which is not usually presented and/or discussed in many physics textbooks, but one which is very interesting:

$$\boxed{\begin{aligned} I_{tot}(t) &= \tfrac{1}{2}A_{10}^2\left[1 + \cos^2 2\left(\omega_1(t)t\right)\right] + \tfrac{1}{2}A_{20}^2\left[1 + \cos^2 2\left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)\right] \\ &+ A_{10}A_{20}\left[\cos\left(\Omega_{21}(t)t + \Delta\varphi_{21}(t)\right) + \cos\left(\Delta\omega_{21}(t)t - \Delta\varphi_{21}(t)\right)\right] \end{aligned}}$$

This latter formula shows that there are:

©Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois 2002-2017. All rights reserved.