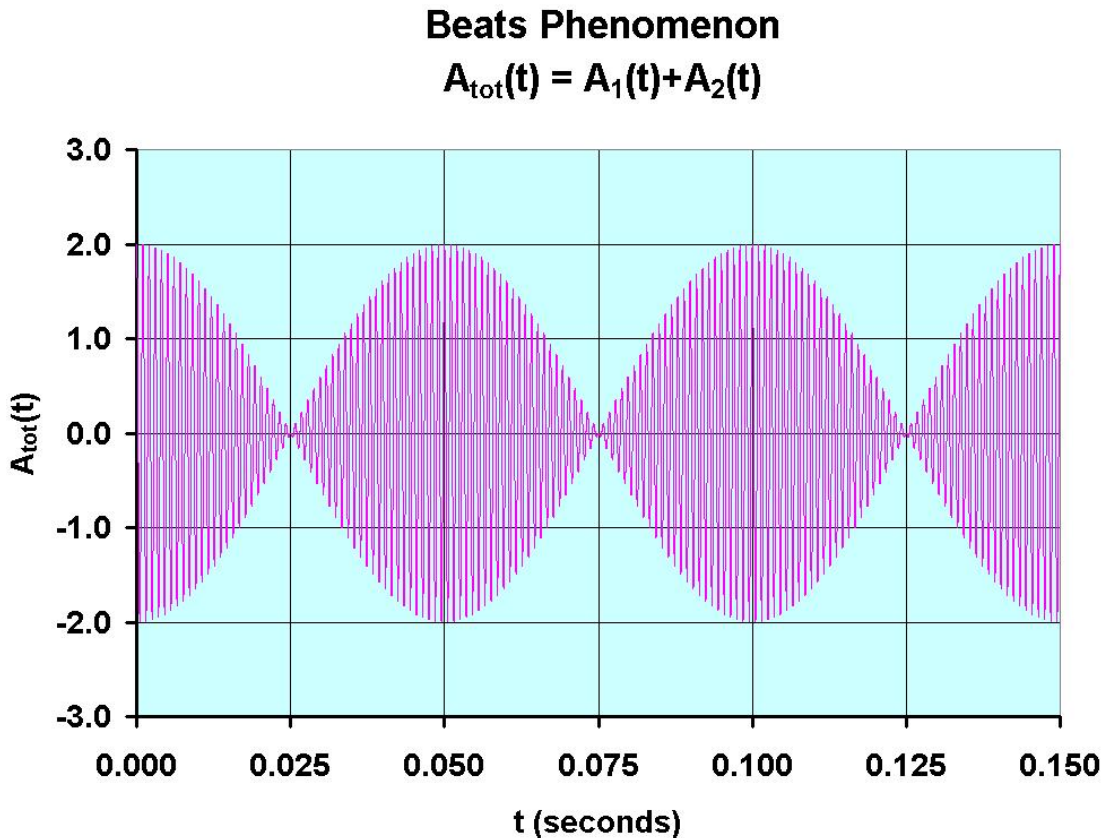


The phase of the total amplitude  $A_{\text{tot}}(t)$  relative to that of the first amplitude  $A_1(t)$ , at an arbitrary time  $t$  is  $\Delta\psi(t)$  and is obtained from the projections of the total amplitude phasor  $A_{\text{tot}}(t)$  onto the  $y$ - and  $x$ - axes of the 2-D phasor plane:

$$\tan \Delta\psi = \frac{A_2(t) \cos(\omega_2(t)t + \Delta\phi_{21}(t)) \sin \Delta\phi_{21}(t)}{A_1(t) \cos(\omega_1(t)t) + A_2(t) \cos(\omega_2(t)t + \Delta\phi_{21}(t)) \cos \Delta\phi_{21}(t)}$$

The total amplitude  $A_{\text{tot}}(t) = A_1(t) + A_2(t)$  vs. time  $t$  is shown in the figure below, for time-independent/constant frequencies of  $f_1 = 1000 \text{ Hz}$  and  $f_2 = 980 \text{ Hz}$ , equal amplitudes of unit strength  $A_{10} = A_{20} = 1.0$  and zero relative initial phase  $\Delta\phi_{21} = 0.0$



Clearly, the beats phenomenon can be seen in the above waveform of total amplitude  $A_{\text{tot}}(t) = A_1(t) + A_2(t)$  vs. time  $t$ . When  $A_{\text{tot}}(t) = 0$ , we have complete destructive interference of the two individual amplitudes – *i.e.* the 2<sup>nd</sup> amplitude is  $180^\circ$  out of phase relative to the first. When  $A_{\text{tot}}(t) = 2$ , we have complete constructive interference of the two amplitudes – the two individual amplitudes are exactly in phase with each other. More on this, below...

From the above graph, it is also obvious that the beat period  $\tau_{\text{beat}} = 1/f_{\text{beat}} = 0.050 \text{ sec} = 1/20^{\text{th}}$  sec, corresponding to a beat frequency of  $f_{\text{beat}} = 1/\tau_{\text{beat}} = 20 \text{ Hz}$ , which is simply the (absolute value of the) frequency difference  $f_{\text{beat}} \equiv |f_1 - f_2|$  between  $f_1 = 1000 \text{ Hz}$  and  $f_2 = 980 \text{ Hz}$ . Thus, the beat period  $\tau_{\text{beat}} = 1/f_{\text{beat}} = 1/|f_1 - f_2|$ . When  $f_1 = f_2$ , the beat period becomes infinitely long, and no beats are heard.