<u>Beats Phenomenon</u>:

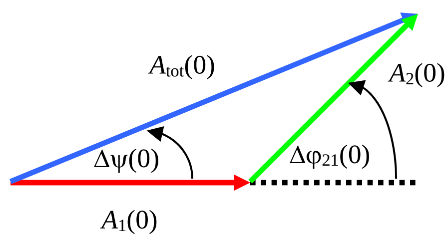
Beats is the phenomenon of interference with 2 (or more) signals of *approximately* the same, but *NOT* identical frequency, *i.e.* $f_1 \sim f_2$.

Linearly superpose (*i.e.* add) two "generic" signals with amplitudes $A_1(t)$ and $A_2(t)$, and which have similar/comparable frequencies, $\omega_2(t) = 2\pi f_2(t) \sim \omega_1(t) = 2\pi f_1(t)$, with instantaneous phase of the second signal relative to the first of $\Delta\varphi_{21}(t)$:

$$A_1(t) = A_{10}\cos\left(\omega_1(t)t\right) \qquad A_2(t) = A_{20}\cos\left(\omega_2(t)t + \Delta\phi_{21}(t)\right)$$
$$A_{tot}(t) = A_1(t) + A_2(t) = A_{10}\cos\left(\omega_1(t)t\right) + A_{20}\cos\left(\omega_2(t)t + \Delta\phi_{21}(t)\right)$$

Note that at the amplitude level, there is nothing explicitly overt and/or obvious in the above mathematical expression for the overall/total/resultant amplitude, $A_{tot}(t)$ that *easily* explains the phenomenon of beats associated with adding together two signals that have comparable amplitudes and frequencies. From the above formula, clearly the total waveform simply consists of two individual waveforms, one with slightly different frequency than the other.

However, let us consider the (instantaneous) <u>phasor</u> <u>relationship</u> between the individual amplitudes for the two signals, $A_1(t)$ and $A_2(t)$ respectively. Their relative initial phase difference at time t = 0 is $\Delta\varphi_{21}(t=0)$ and the resultant/total amplitude, $A_{tot}(t=0)$ is shown in the figure below, for time, t = 0:



From the *<u>law of cosines</u>*, the magnitude of the total amplitude, $A_{tot}(t)$ at an arbitrary time t is obtained from the following:

$$A_{tot}^2(t) = A_1^2(t) + A_2^2(t) - 2A_1(t)A_2(t)\cos\left[\pi - \left((\omega_2(t) - \omega_1(t))t + \Delta\phi_{21}(t)\right)\right]$$
$$A_{tot}^2(t) = A_1^2(t) + A_2^2(t) + 2A_1(t)A_2(t)\cos\left((\omega_2(t) - \omega_1(t))t + \Delta\phi_{21}(t)\right)$$

Thus:

$$A_{tot}(t) = \sqrt{A_1^2(t) + A_2^2(t) + 2A_1(t)A_2(t)\cos\left((\omega_1(t) - \omega_2(t))t + \Delta\phi_{21}(t)\right)}$$
$$= \sqrt{\begin{array}{l} A_{10}^2\cos^2\left(\omega_1(t)t\right) + A_{20}^2\cos^2\left(\omega_2(t)t\right) \\ +2A_{10}A_{20}\cos\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t + \Delta\phi_{21}(t)\right)\cos\left((\omega_2(t) - \omega_1(t))t + \Delta\phi_{21}(t)\right) \end{array}}$$

For <u>equal</u> amplitudes $A_{10} = A_{20} = A_0$, zero relative initial phase $\Delta\varphi_{21} = 0$ and constant (i.e. time-independent) frequencies, ω_2 and ω_1, this expression simplifies to:

$$A_{tot}(t) = A_0\sqrt{\cos^2\omega_1 t + \cos^2\omega_2 t + 2\cos\omega_1 t\cos\omega_2 t\cos\left((\omega_2 - \omega_1)t\right)}$$