Then for ***small*** pressure variations $\left(p = P - P_{atm} = \Delta P \ll P_{atm}\right)$: $\Rightarrow \left(\Delta P/P\right) = -\gamma\left(\Delta V/V\right)$

However, from above: $B \equiv -\dfrac{\Delta P}{\left(\Delta V/V\right)}$. Thus, we see that: $B = \gamma P \simeq \gamma P_{atm}$ for $p = P - P_{atm} \ll P_{atm}$.

For 1-D traveling plane waves: $p = \Delta P = -B\left(\Delta V/V\right) = -B\dfrac{\partial \xi_z\left(z,t\right)}{\partial z}$.

Thus, we see that there is a relation between overpressure $p\left(z,t\right)$ and the local *slope* (*i.e.* the spatial gradient) of the longitudinal displacement $\partial\xi\left(z,t\right)/\partial z$:

If longitudinal displacement: $\boxed{\xi_z(z,t) = \xi_o\cos\left(\omega t - kz\right)}$ then: $p\left(z,t\right) = -B\,\partial\xi_z\left(z,t\right)/\partial z$ gives:

$$\boxed{p\left(z,t\right) = p\left(z,t\right) - P_{atm} = \Delta P\left(z,t\right) = -B\frac{\partial \xi_z\left(z,t\right)}{\partial z} = -Bk\xi_o\sin\left(\omega t - kz\right) = p_o\sin\left(\omega t - kz\right)}$$

Hence, we see that the over-pressure $p\left(z,t\right)$ and the longitudinal displacement of air molecules from their equilibrium positions $Z(z,t)$ are 90° out-of-phase with each other.

The (mean, or avg.) longitudinal <u>speed</u> of air molecules $u_z(z,t)$ is the time rate of change of the (mean, or avg.) longitudinal displacement of air molecules, *i.e*. the time derivative $\partial\xi_z\left(z,t\right)/\partial t$.

Thus, for a harmonic/sinusoidal sound wave in air, the instantaneous particle velocity is:

$$\boxed{u_z\left(z,t\right) = \partial\xi_z\left(z,t\right)/\partial t = -\omega\xi_o\sin\left(\omega t - kz\right) = u_o\sin\left(\omega t - kz\right)}$$

Thus, we also see that the longitudinal speed of air molecules $u_z\left(z,t\right)$ and the overpressure $p\left(z,t\right)$ are <u>in-phase</u> with each other for sound waves propagating in "free" air:

$$\boxed{p\left(z,t\right) = p_o\sin\left(\omega t - kz\right) = -B\frac{\partial\xi\left(z,t\right)}{\partial z} = -Bk\xi_0\sin\left(kz - \omega t\right) = -\frac{Bk}{\omega}\omega\xi_0\sin\left(kz - \omega t\right) = \frac{Bk}{\omega}u_z\left(z,t\right)}$$

A medium has a so-called ***<u>characteristic specific acoustic impedance</u>*** $z_a\left(\vec{r}\right)$ associated with it at the listener point $\vec{r}$ – which physically is a measure of how easy (or difficult) it is for (acoustic) energy to ***<u>flow</u>*** from one point to another in the medium. For ***<u>longitudinal</u>*** traveling acoustic plane waves propagating ***freely*** in a gas, the ***characteristic longitudinal specific acoustic impedance*** $z_a^{\parallel}\left(\vec{r}\right)$ is defined as (using $v = \omega/k = f\lambda$, and $B = \rho v^2$ {see below…}):

$\boxed{z_a^{\parallel}\left(\vec{r}\right) \equiv p\left(\vec{r}\right)/u_z\left(\vec{r}\right) = Bk/\omega = B/v = \rho v}$ (*SI* units of $z_a\left(\vec{r}\right)$: *Pa-s/m* = acoustic Ohms).

aka ***<u>Rayls</u>***, in honor of Lord Rayleigh (John William Strutt)

The ***characteristic longitudinal specific acoustic impedance*** for ***air*** is:

$\boxed{z_a^{\parallel,air}\left(\vec{r}\right) = \rho_o^{air}v_o^{air} = 1.204\,kg/m^3\cdot 345\,m/s \simeq 415\ Pa\text{-}s/m = 415\,\Omega_{ac} = 415\ Rayls}$ @ NTP.