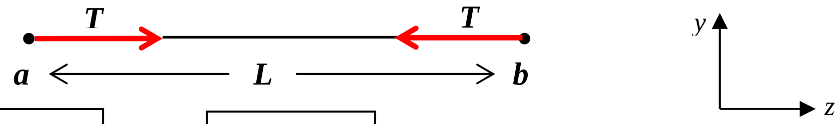
The longitudinal speed of propagation v of ***transverse*** displacement traveling waves, where the displacement (from equilibrium position) is *e.g.* in the y-direction, perpendicular (*i.e.* transverse) to the direction of propagation, *e.g.* in the $\pm z$-direction is mathematically described by:

$\boxed{Y(z,t) = Y_o \sin(\omega t \mp kz)}$ for a ***transverse*** traveling plane wave propagating in the $\pm z$-direction.

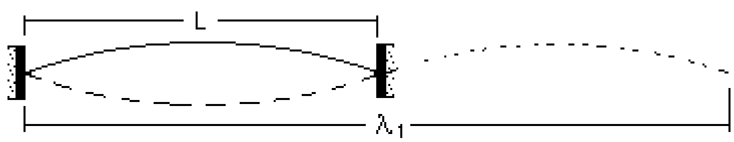
For ***transverse*** waves propagating on a ***stretched string*** having tension, T (*n.b.* the *SI* / metric units of tension T (= force) is Newtons, $N = kg\text{-}m/s^2$), the string has mass M and length L, the longitudinal speed of propagation of transverse traveling waves on a string is given by:



$\boxed{v = \sqrt{T/\mu}}$ where: $\boxed{\mu = M/L}$ = mass per unit length of string (*SI* units: kg/m)

Example: Tension and Transverse {Standing} Waves on the High-E String of a Guitar:

n.b. A standing wave = superposition of two traveling waves propagating in opposite directions!



$f_{Hi\text{-}E}$ = 332 Hz for open Hi-E string.

The high-E string on a guitar has diameter, D = 0.009" (~230μm) $\quad (0.001'' = 1/1000'' = 25.4\mu m)$

L = string length = 63.5 cm = 0.635m (= 25.0" = scale length *e.g.* of a Fender electric guitar)

ρ = density of string = 7.9 gms/cm^3 = 7900 kg/m^3 for ***steel***.

$A_\perp$ = cross-sectional area of string = $\pi R^2 = \pi(D^2/4) = 4.104\times10^{-3}\ cm^2 = 4.104\times10^{-9}\ m^2$.

V = volume of string $= A_\perp \cdot L = \pi R^2 \cdot L = \pi (D/2)^2 \cdot L = \pi D^2 L/4 = 0.026\, cm^3 = 0.026\times10^{-6}\, m^3$

Mass of string: $M = \rho V = \rho(A_\perp \cdot L) = \rho(\pi D^2 L/4) = 0.206\ gms = 0.206\times10^{-3}\,kg$

$$\mu = \frac{M}{L} = \frac{\rho V}{L} = \frac{\rho(A_\perp \cdot \not{L})}{\not{L}} = \rho \cdot A_\perp = \frac{0.206\times10^{-3}\,kg}{0.635m} = 3.242\times10^{-4}\ kg/m$$

Now $f_{Hi\text{-}E}$ = 332 Hz, and $\lambda_{Hi\text{-}E} = 2L = 2\times0.635\ m = 1.27\ m$ (see above pix, for fundamental)

Thus, the longitudinal speed of propagation of transverse traveling waves on the Hi-E string of a guitar is:

$$v_{Hi\text{-}E} = f_{Hi\text{-}E} * \lambda_{Hi\text{-}E} = 332\ Hz * 1.27\ m = 421.6\ m/s$$