However, since:
$$
\overline{v_o = -\omega x_o}
$$
 and: $\omega = \sqrt{\frac{k}{M}}$ thus: $\overline{k = M \omega^2}$ Hence, we see that:
\n
$$
P.E.(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kx_o^2 \cos^2(\omega t)
$$
\n
$$
K.E.(t) = \frac{1}{2}Mv^2(t) = \frac{1}{2}Mv_o^2 \sin^2(\omega t) = \frac{1}{2}M\omega^2 x_o^2 \sin^2(\omega t) = \frac{1}{2}kx_o^2 \sin^2(\omega t)
$$
\nLet us define: $E_o = \frac{1}{2}kx_o^2 = \frac{1}{2}M\omega^2 x_o^2$ $\frac{\text{Then:}}{\text{Then:}} \begin{bmatrix} P.E.(t) = E_o \cos^2(\omega t) \\ K.E.(t) = E_o \sin^2(\omega t) \end{bmatrix}$

We define the *total* energy, $E_{tot}(t)$ as the sum of instantaneous potential + kinetic energies:

$$
E_{tot}(t) = P.E.(t) + K.E.(t) = E_o \cos^2(\omega t) + E_o \sin^2(\omega t) = E_o \left\{ \cos^2(\omega t) + \sin^2(\omega t) \right\}
$$

Using the trigonometric identity $1 = \cos^2 x + \sin^2 x$ we see that:

$$
E_{\text{Tot}}(t) = E_o = \frac{1}{2} k x_o^2 = \frac{1}{2} M \omega^2 x_o^2 = \frac{\text{constant}}{\text{constant}} (-0), \text{ independent of time!}
$$

Thus, the total energy in (spring + mass) system *is* constant – due to <u>conservation of energy</u>!!

Graphs of $P.E.(t)$, $K.E.(t)$, and $E_{tot}(t)$ vs. time (all normalized to E_o) are shown in the figure below:

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