However, since: $v_o = -\omega x_o$ and: $\omega = \sqrt{\frac{k}{M}}$ thus: $k = M\omega^2$ Hence, we see that:

$$P.E.(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kx_o^2\cos^2(\omega t)$$

$$K.E.(t) = \frac{1}{2}Mv^2(t) = \frac{1}{2}Mv_o^2\sin^2(\omega t) = \frac{1}{2}M\omega^2 x_o^2\sin^2(\omega t) = \frac{1}{2}kx_o^2\sin^2(\omega t)$$

Let us define: $E_o \equiv \frac{1}{2}kx_o^2 = \frac{1}{2}M\omega^2 x_o^2$ Then: $P.E.(t) = E_o\cos^2(\omega t)$, $K.E.(t) = E_o\sin^2(\omega t)$

We define the *total* energy, $E_{tot}(t)$ as the sum of instantaneous potential + kinetic energies:

$$E_{tot}(t) = P.E.(t) + K.E.(t) = E_o\cos^2(\omega t) + E_o\sin^2(\omega t) = E_o\left\{\cos^2(\omega t) + \sin^2(\omega t)\right\}$$

Using the trigonometric identity $1 = \cos^2 x + \sin^2 x$ we see that:

$$E_{Tot}(t) = E_o = \frac{1}{2}kx_o^2 = \frac{1}{2}M\omega^2 x_o^2 = \text{constant } (>0), \text{ independent of time!}$$

Thus, the total energy in (spring + mass) system *is* constant – due to conservation of energy!!

Graphs of $P.E.(t)$, $K.E.(t)$, and $E_{tot}(t)$ vs. time (all normalized to E_o) are shown in the figure below: