Once the mass *M* has been set in motion, Newton's 2<sup>nd</sup> Law tells us that:  $F(t) = -kx(t) = Ma(t)$ However:  $\left| x(t) = x_o \cos(2\pi ft) \right| = x_o \cos(\omega t)$ And from above, we also know that:  $\begin{vmatrix} a_{0} = \omega^{2}x_{0} = (2\pi f)^{2}x_{0} \end{vmatrix}$   $\therefore \begin{vmatrix} -kx_{0} = -\omega^{2}x_{0} \end{vmatrix}$  $\therefore$   $-kx_0 = -\omega^2 Mx_0$ Thus, the frequency *f* and angular frequency  $\omega$  of oscillation of the mass *M* on the spring are:  $\left| \begin{array}{c} t \end{array} \right|$   $\left| \begin{array}{c} k \end{array} \right|$  Cycles per second, or Hz  $\left| \begin{array}{c} \end{array} \right|$  and  $\left| \omega = 2\pi f = \sqrt{\frac{k}{n}} \right|$  (*radians/sec*) The period of oscillation  $\tau$  of the mass *M* on the spring is:  $\left( \frac{\tau}{\tau} - \frac{1}{\tau} - 2\pi \right) \frac{M}{M}$  (*seconds*) Note also that since the instantaneous acceleration  $a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2}$  $dv(t)$   $d^2x(t)$  $a(t) = \frac{dv(t)}{dt} = \frac{dv(t)}{dt^2}$ , then we can write Newton's  $2<sup>nd</sup>$  law for this system as a differential equation:  $F(t) = -kx(t) = Ma(t) \implies -kx(t) = M \frac{d^2x(t)}{dt^2}$  $d^2x(t)$  $kx(t) = M$  $-kx(t) = M \frac{d^2x(t)}{dt^2}$  or:  $M \frac{d^2x(t)}{dt^2} + kx(t)$  $\frac{y}{2} + kx(t) = 0$  $d^2x(t)$  $M \frac{d^{n}x(t)}{dt^{2}} + kx(t)$ *dt*  $kx(t) = 0$  or:  $M \ddot{x}(t) + kx(t) = 0$  $d^2x(t)$ and:  $a(t) = -a_0 \cos(2\pi ft) = -a_0 \cos(\omega t)$  $f = \frac{1}{2\pi} \sqrt{\frac{k}{M}}$  Cycles per second, or *Hz* and  $\omega = 2\pi f = \sqrt{\frac{k}{M}}$  $\omega = 2\pi f = \sqrt{\frac{k}{h}}$ *k M f*  $\tau = \frac{1}{2} = 2\pi$ 

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 $\ddot{x}(t) = \frac{d^{2}x(t)}{dt^{2}}$ .

which is a linear, homogenous  $2<sup>nd</sup>$ -order differential equation, and where:

The instantaneous potential energy *stored* in the stretched/compressed spring is:

$$
P.E.(t) = \frac{1}{2}kx^{2}(t) \quad (Joules)
$$

The instantaneous kinetic energy associated with the *moving* mass, *M* is:

$$
K.E.(t) = \frac{1}{2}Mv^{2}(t)
$$
 (Joules)

The potential energy of the spring and the kinetic energy of the moving mass are both time dependent:

$$
P.E.(t) = \frac{1}{2}kx^{2}(t) = \frac{1}{2}kx_{o}^{2}\cos^{2}(\omega t) \ge 0
$$
  

$$
K.E.(t) = \frac{1}{2}Mv^{2}(t) = \frac{1}{2}Mv_{o}^{2}\sin^{2}(\omega t) \ge 0
$$

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