$$
a(t) = \frac{\Delta v(t)}{\Delta t} = \frac{dv(t)}{dt} = \text{total derivative of } v(t) \text{ with respect to time, } t.
$$
  

$$
a(t) = \frac{d}{dt} \left( v(t) \right) = \frac{d}{dt} \left[ v_o \sin \left( 2\pi f t \right) \right] = 2\pi f \ v_o \cos \left( 2\pi f t \right) = \omega v_o \cos \left( \omega t \right) \equiv a_o \cos \left( \omega t \right)
$$
  
We see that: 
$$
a_o = \omega v_o = 2\pi f \ v_o \quad \text{but:} \quad v_o = -\omega x_o = -2\pi f \ x_o \quad \therefore \quad a_o = -\omega^2 x_o = -\left( 2\pi f \right)^2 x_o
$$

*i.e.* the acceleration amplitude,  $A<sub>o</sub>$  = max acceleration is related to the displacement amplitude, *Xo* by this formula for *harmonic* motion.

Instantaneous Horizontal Accel. of the Moving Mass:  $\left(m/s^2\right)$  $(meters/sec<sup>2</sup>)$  $(meters/sec<sup>2</sup>)$  acceleration  $\Box$  frequency of oscillation  $\frac{\text{amplitude}}{\text{m/s}^2}$  (cycles per second = Hertz)  $a(t) = a_o \cos(2\pi ft) = a_o \cos(\omega t)$ 

The time dependence of the longitudinal position,  $x(t)$  (*i.e.* displacement of the mass from its equilibrium position) *vs*. time, *t* and longitudinal speed of the mass,  $v(t)$  *vs*. time, *t* and longitudinal acceleration  $a(t)$  vs. time, *t* are shown in the figure below; note that each has been normalized to their respective amplitudes (note also the phase relation between  $x(t)$ ,  $v(t)$  and  $a(t)$ ):



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