$$a(t) = \frac{\Delta v(t)}{\Delta t} = \frac{dv(t)}{dt} = \text{total derivative of } v(t) \text{ with respect to time, } t.$$

$$a(t) = \frac{d}{dt} (v(t)) = \frac{d}{dt} \Big[ v_o \sin(2\pi ft) \Big] = 2\pi f v_o \cos(2\pi ft) = \omega v_o \cos(\omega t) \equiv a_o \cos(\omega t)$$

$$\underline{We \text{ see that:}} \quad \boxed{a_o = \omega v_o = 2\pi f v_o} \quad \underline{but:} \quad \boxed{v_o = -\omega x_o = -2\pi f x_o} \quad \therefore \quad \boxed{a_o = -\omega^2 x_o = -(2\pi f)^2 x_o}$$

$$i \text{ on the constant of } u \text{ and } u \text{ on the constant of } u \text{ on th$$

*i.e.* the acceleration amplitude,  $A_o = \max$  acceleration is related to the displacement amplitude,  $X_o$  by this formula for *harmonic* motion.

Instantaneous Horizontal Accel. of the Moving Mass:  $a(t) = a_o \cos(2\pi ft) = a_o \cos(\omega t) \qquad (m/s^2)$ (meters/sec<sup>2</sup>) acceleration amplitude (m/s<sup>2</sup>) (cycles per second = Hertz)

The time dependence of the longitudinal position, x(t) (*i.e.* displacement of the mass from its equilibrium position) *vs*. time, *t* and longitudinal speed of the mass, v(t) *vs*. time, *t* and longitudinal acceleration a(t) vs. time, *t* are shown in the figure below; note that each has been normalized to their respective amplitudes (note also the phase relation between x(t), v(t) and a(t)):



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