If there is no friction, and the mass M is horizontally displaced from its equilibrium ($x = 0$) position by pulling on it to the right, as shown in the above figure, the force necessary to accomplish this is $\boxed{F_I = +kx_o}$ (Hooke's Law), where $k > 0$ is the so-called "spring constant" of the spring (k has metric units of Newtons/meter) and x_o = the initial displacement of the mass M from its $x = 0$ equilibrium position.

At time $t = 0$ the mass is released. At that instant, the only force acting on the mass is due to the {horizontal} restoring force of the spring: $\boxed{F_S(t=0) = -kx_o = -F_I}$. However, from Newton's 2nd Law: $F(t) = Ma(t)$, therefore at time $t = 0$: $\boxed{F_S(t=0) = -kx_o = Ma(t=0)}$.

As time progresses the mass M oscillates horizontally back and forth about its $x = 0$ equilibrium position, exhibiting sinusoidal/harmonic motion. Mathematically, the time-dependence of this horizontal sinusoidal/harmonic motion is described by:

Longitudinal displacement from equilibrium: $\boxed{x(t) = x_o \cos(2\pi f t) = x_o \cos(\omega t)}$ (m)

(meters) displacement amplitude (meters) frequency of oscillation (cycles per second = Hertz) cps Hz

Omega: $\boxed{\omega \equiv 2\pi f = \text{angular frequency (units = radians per second)}}$

Period of oscillation: $\boxed{\tau \equiv \frac{1}{f} = \frac{2\pi}{\omega}}$ (seconds)

The instantaneous horizontal speed of the moving mass $v(t)$ with time t is defined as the time rate of change of the horizontal position (longitudinal displacement) of the moving mass with time t, physically, $v(t)$ is the instantaneous local slope of the $x(t)$ vs. t graph at time t:

$$v(t) = \frac{\Delta x(t)}{\Delta t} = \frac{dx(t)}{dt} = \text{ total derivative of } x \text{ with respect to time, } t \text{ \{since 1-D partial => total\}}.$$

$$v(t) = \frac{d}{dt}(x(t)) = \frac{d}{dt}\left[x_o \cos(2\pi f t)\right] = -2\pi f\, x_o \sin(2\pi f t) = -\omega x_o \sin(\omega t) \equiv v_o \sin(\omega t)$$

We see that: $\boxed{v_o = -\omega x_o = -2\pi f\, x_o}$

– sign defines the ***phase relation*** between velocity $v(t)$ ***relative*** to displacement $x(t)$.

i.e. the speed "amplitude", v_o = max speed is related to the displacement amplitude, x_o by this formula for ***harmonic*** motion.

Instantaneous Horizontal Speed of the Moving Mass: $\boxed{v(t) = v_o \sin(2\pi f t) = v_o \sin(\omega t)}$ (m/s)

(meters/sec) speed amplitude (m/s) frequency of oscillation (cycles per second = Hertz)

The instantaneous horizontal acceleration of the moving mass $a(t)$ with time t is defined as the time rate of change of the horizontal speed of the moving mass with time t, physically, $a(t)$ is the instantaneous local slope of the $v(t)$ *vs.* t graph at time t: