

Amplitude (Volume/Loudness) Fluctuations in Human Music:

The instantaneous acoustic power $P_{ac}(t)$ output *e.g.* from a loudspeaker is related to the instantaneous electrical power input to the loudspeaker $P_{em}(t)$ by $P_{ac}(t) = \varepsilon_{ls} P_{em}(t)$ where ε_{ls} is the loudspeaker's efficiency for converting electrical power into acoustical power, typically $\sim O(1\text{--}few\ \%)$. Using Ohm's law, the instantaneous electrical power input to the loudspeaker is proportional to the **square** of the instantaneous voltage $V(t)$ across the terminals of the loudspeaker: $P_{em}(t) \simeq V^2(t)/R_{ls}$, where R_{ls} is the resistance of the loudspeaker.

The on-axis, **direct** sound pressure level associated with the sound coming from the loudspeaker, heard by a listener located a distance r_{\perp} away from, but along the axis of the loudspeaker is:

$$SPL_{direct}(r_{\perp}, t) = L_p^{direct}(r_{\perp}, t) = L_{Pwr}(t) + 10 \log_{10}(Q/4\pi r_{\perp}^2) \text{ (dB)}$$

where the loudness level $L_{Pwr}(t) \equiv 10 \log_{10}(P_{ac}(t)/P_{ac}^o) \text{ (dB)}$, the **reference** acoustic power level $P_{em}^o \equiv 10^{-12} \text{ Watts}$ and Q is the directivity factor of the loudspeaker. Thus, we see that:

$$\begin{aligned} SPL_{direct}(r_{\perp}, t) &= 10 \log_{10}(P_{ac}(t)/P_{ac}^o) + 10 \log_{10}(Q/4\pi r_{\perp}^2) \text{ (dB)} \\ &= 10 \log_{10}(\varepsilon_{ls} P_{em}(t)/P_{ac}^o) + 10 \log_{10}(Q/4\pi r_{\perp}^2) \text{ (dB)} \\ &= 10 \log_{10}(\varepsilon_{ls} V^2(t)/R_{ls} P_{ac}^o) + 10 \log_{10}(Q/4\pi r_{\perp}^2) \text{ (dB)} \end{aligned}$$

Recall that in a free-field acoustics situation, the Loudness = Sound **Intensity** Level:

$$L_t^{direct}(r_{\perp}, t) \equiv 10 \log_{10}(I(r_{\perp}, t)/I_o) \approx L_p^{direct}(r_{\perp}, t) \equiv 20 \log_{10}(p(r_{\perp}, t)/p_o) \text{ to within } \sim 0.1 \text{ dB.}$$

Thus, we see that Loudness is proportional to {the base-10 log} of $V^2(t)$. Hence, the moment-to-moment fluctuations in the Loudness associated with human music can be obtained *e.g.* by squaring the instantaneous electrical voltage associated with a music signal and obtaining the corresponding PSD function $S_{V^2}(f)$ associated with $V^2(t)$, as shown below in the bottom left & right figures 2 & 3, taken from the seminal paper: "1/f Noise in Music: Music from 1/f Noise", R.F. Voss and J. Clarke, J. Acoust. Soc. Am. **63**, p. 258-263 (1978). The instantaneous music voltage signal $V(t)$ was first band-pass filtered in the 100 – 10 KHz frequency range, squared and then sent through a 20 Hz low-pass filter to observe the moment-to-moment Loudness correlations in human music. The log-log plot of the audio PSD function $S_{V^2}(f)$ in the bottom right-hand figure 3 clearly shows $1/f^1$ loudness fluctuations associated with Bach's 1st Brandenburg Concerto. Figure 4 shows the audio PSD function $S_{V^2}(f)$ associated with audio signals from different radio stations. Figure 5 shows the audio PSD function $S_{V^2}(f)$ associated with different musical pieces/musical composers, both plots clearly show $1/f^1$ loudness fluctuations!