

Another way to characterize the average behavior of $V(t)$ is via the use of the time-domain auto-correlation (*aka* self-correlation) function $\langle V(t)V(t+\Delta t) \rangle$, which is a measure of how the fluctuating quantities $V(t)$ and $V(t+\Delta t)$ are related to each other as a function of the delay time difference $\Delta t \equiv t_2 - t_1$. For a stationary (*i.e.* time-independent) process, the autocorrelation function $\langle V(t)V(t+\Delta t) \rangle$ is independent of t and depends only on the time difference Δt .

The frequency-domain PSD function $S_V(f)$ and time-domain auto-correlation function $\langle V(t)V(t+\Delta t) \rangle$ are related to each other by the Wiener-Khintchine relations:

$$\langle V(t)V(t+\Delta t) \rangle = \int_{f=0}^{f=\infty} S_V(f) \cos(2\pi f \Delta t) df$$

and:

$$S_V(f) = 4 \int_{\Delta t=0}^{\Delta t=\infty} \langle V(t)V(t+\Delta t) \rangle \cos(2\pi f \Delta t) d(\Delta t)$$

Many (but by no means all) fluctuating quantities $V(t)$ can be characterized by a **single** correlation time constant τ_c . The classic example of Brownian “random walk” motion of pollen grains in water **is** characterized by a single time constant τ_c , the mean time between successive collisions. The fluctuating quantity $V(t)$ is thus **correlated** with $V(t+\Delta t)$ for **short** time differences $\Delta t \ll \tau_c$, but is **independent** of $V(t+\Delta t)$ for **long** time differences $\Delta t \gg \tau_c$. This in turn implies that the PSD function $S_V(f)$ is “white” (*i.e.* flat, independent of frequency) in the frequency range over which $V(t)$ is independent, *i.e.* $f = 1/\Delta t \ll 1/(2\pi\tau_c)$. The PSD function $S_V(f)$ decreases rapidly with increasing frequency (typically as $1/f^2$) in the frequency range $f = 1/\Delta t \gg 1/(2\pi\tau_c)$ over which $V(t)$ is correlated with $V(t+\Delta t)$. Hence, a fluctuating quantity $V(t)$ with *e.g.* a $1/f$ PSD function $S_V(f)$ **cannot** be characterized by a **single** correlation time constant – a $1/f$ PSD function $S_V(f)$ instead implies correlations in $V(t)$ over **all** time scales that correspond to the frequency range for which the PSD function $S_V(\omega)$ exhibits the $1/f$ behavior. Note that in general, the negative slope of $S_V(f)$ implies some degree of correlation. A steep (shallow) slope of $S_V(f)$ implies a strong (weak) degree of correlation, respectively. Hence a fluctuating quantity $V(t)$ with a Brownian $1/f^2$ PSD function $S_V(f)$ is strongly correlated, whereas one with a “white noise” $1/f^0$ (*i.e.* flat) PSD function $S_V(f)$ has **no** temporal correlations.