Thus, the ***frequency-domain real/in-phase*** and ***imaginary/90º-out-of-phase/quadrature*** *RMS* voltage amplitude components output from the *LIA* for the pressure mic located at *x* = *d* will be:

$V_{o_{RMS}}^{Sig}\cos\varphi_S$ and: $V_{o_{RMS}}^{Sig}\sin\varphi_S$, respectively.

Thus, the ***frequency-domain real/in-phase*** and ***imaginary/90º-out-of-phase/quadrature*** *RMS* components of the complex pressure amplitude $\tilde{p}(x=d,\omega)$ at *x* = *d* are:

$p_o\cos\varphi_S = V_{o_{RMS}}^{Sig}\cos\varphi_S / S_{p\text{-}mic}\ (RMS\ Pascals)$ and:
$p_o\sin\varphi_S = V_{o_{RMS}}^{Sig}\sin\varphi_S / S_{p\text{-}mic}\ (RMS\ Pascals)$, respectively, since: $p_o = V_{o_{RMS}}^{Sig} / S_{p\text{-}mic}$.

Thus, we see that at *x* = *d*, a propagation delay time-induced phase shift of $\varphi_S|_{x=d} = -kd$ results from the fact that it takes a ***finite time*** $\Delta t_{prop} = d/c$ for the a ***free-field*** plane wave to propagate in free air from *x* = 0 to *x* = *d*. Since $k = \omega/c = 2\pi/\lambda$ in free-air, we see that the apparent phase shift at $x = d$ is: $\varphi_S|_{x=d} = -kd = -(\omega/c)\cdot d = -(\omega/c)\cdot(c\Delta t_{prop}) = -\omega\Delta t_{prop}$.

Thus {here}, we see that the phase shift $\varphi_S|_{x=d}$ at $x = d$ is in fact frequency dependent, linearly proportional to the (angular) frequency: $\varphi_S(\omega)|_{x=d} = -k(\omega)d = -\omega\cdot\Delta t_{prop}(\omega)$, becoming more negative with increasing (angular) frequency $\omega = 2\pi f$. See figure below.

While the propagation delay time-induced phase shift effect may initially be perceived as an experimental annoyance, it is actually a physics blessing in disguise!

If the *p*-mic distance *d* from the sound source is known, then a measurement of the ***phase speed*** of sound (the speed at which the phase {*i.e*. the crests/troughs of sound waves} advances) $c_\phi(\omega) \equiv \omega/k(\omega) = f\cdot\lambda(\omega)$ *vs*. frequency *f* can be obtained using:

$$\boxed{c_\phi(\omega) \equiv \omega/k(\omega) = -\left(\omega/\varphi_S(\omega)|_{x=d}\right)\cdot d = -\left(2\pi f/\varphi_S(f)|_{x=d}\right)\cdot d\ \ (m/s)}$$

The ***group speed*** of propagation of sound waves ((the speed at which ***energy/information*** propagates) is defined as: $c_g(\omega) \equiv \left[dk(\omega)/d\omega\right]^{-1}$, which is the {inverse of the} ***local slope*** of the graph of $k(\omega)$ *vs*. ω (at fixed *p*-mic position, *d*). Thus, since $\varphi_S(\omega)|_{x=d} = -k(\omega)d$, then: $d\varphi_S(\omega)|_{x=d}/d\omega = -\left(dk(\omega)/d\omega\right)\cdot d$, hence the ***group speed*** of propagation of sound waves

$$\boxed{c_g(\omega) \equiv \left[dk(\omega)/d\omega\right]^{-1} = d\left[-d\varphi_S(\omega)|_{x=d}/d\omega\right]^{-1}\ (m/s)}$$

For propagation of ***free-field*** monochromatic traveling plane waves, the local slope $dk(\omega)/d\omega = k(\omega)/\omega$, hence $c_g(\omega) = c_\phi(\omega) = \omega/k(\omega)$ in the ***free-field***. In general, this is not the case for an arbitrary sound field, *e.g*. the near *vs*. far zone associated with a point/monopole sound source, or *e.g*. a plane circular piston of radius *a* (an approximation to a loudspeaker).