Next, we can either ***time-average*** the two *PSD* ***product*** signals, or *e.g.* send them through a {very} ***low-pass filter*** (with –3 *dB* corner frequency $\omega_{-3\,dB} \ll 2\omega$). By doing either of these, we eliminate (*i.e.* ***reject***) the time-dependent/oscillatory (2ω) frequency components of the two *PSD* ***product*** signals; hence only the ***time-independent*** (*i.e.* the zero-frequency) components of the two *PSD* ***product*** signals remain:

$$\left\langle V^{PSD_X}(\omega,t)\right\rangle_t = V_{LPF}^{PSD_X}(\omega) = V_{o_{RMS}}^{S}(\omega)\cos\left(\varphi_S(\omega)-\varphi_R\right)$$

and:
$$\left\langle V^{PSD_Y}(\omega,t)\right\rangle_t = V_{LPF}^{PSD_Y}(\omega) = V_{o_{RMS}}^{S}(\omega)\sin\left(\varphi_S(\omega)-\varphi_R\right)$$

If we now set the {absolute} ***reference*** signal's phase $\varphi_R \equiv 0$, since it ***is*** the ***reference*** phase {*n.b.* we can also eliminate φ_R simply by re-defining the zero of time}, then the two ***time-averaged/LPF 'd time-independent*** *PSD* ***product*** signals simplify further to:

$$\left\langle V^{PSD_X}(\omega,t)\right\rangle_t = V_{LPF}^{PSD_X}(\omega) = V_{o_{RMS}}^{S}(\omega)\cos\varphi_S(\omega)$$

and:
$$\left\langle V^{PSD_Y}(\omega,t)\right\rangle_t = V_{LPF}^{PSD_Y}(\omega) = V_{o_{RMS}}^{S}(\omega)\sin\varphi_S(\omega)$$

Thus, we see that a dual-channel *LIA* enables us to measure the *RMS* values of the ***real/in-phase*** and ***imaginary/90º-out-of-phase/quadrature*** components of a ***periodic*** complex signal ***amplitude***, relative to a ***reference*** sine-wave signal – *i.e.* obtain the ***frequency-domain*** representation of the *RMS* complex harmonic amplitude of a generic "black-box" output response signal, phase-referenced to the input ***reference*** signal:

$$\tilde{V}_{RMS}^{Sig}(\omega) = V_{o_{RMS}}^{Sig}(\omega)\left[\cos\varphi_S(\omega) + i\sin\varphi_S(\omega)\right] = V_{o_{RMS}}^{Sig}(\omega)\cdot e^{i\varphi_S(\omega)}$$

The ***time-domain*** representation of the *RMS* complex harmonic amplitude associated with the generic "black-box" output response signalm phase-referenced to the input ***reference*** signal is:

$$\tilde{V}_{RMS}^{Sig}(\omega,t) = \tilde{V}_{RMS}^{Sig}(\omega)\cdot e^{i\omega t} = \left[\tilde{V}_{o_{RMS}}^{Sig}(\omega)\cdot e^{i\varphi_S(\omega)}\right]\cdot e^{i\omega t} = V_{o_{RMS}}^{Sig}(\omega)\cdot e^{i\left(\omega t+\varphi_S(\omega)\right)}$$

The dual-channel *LIA* is an extremely useful, versatile, sensitive and powerful device. It is routinely used in all sorts of applications involving the measurement and analysis of periodic/pure-tone/single-frequency complex signals.

An important aspect of a *LIA* is that it is inherently a narrow-band frequency device. One sets the bandwidth sensitivity of the *LIA* by specifying its ***settling time constant*** τ (*sec*), which, from the uncertainty relation $\Delta f\,\Delta t = \Delta f\cdot\tau = 1$, sets the *LIA*'s bandwidth: $BW = \Delta f = 1/\tau$. When the ***reference*** signal's frequency changes abruptly $f \rightarrow f'$, the system's output ***response*** signal (input to the *LIA*) ***also*** changes abruptly. It is therefore good experimental practice to wait at least Δt ~ 5 time constants in order to allow the *X*, *Y* (***real/in-phase*** and ***imaginary/90-degree-out-of-phase***) outputs of the *LIA* to settle to within $1-e^{-\Delta t/\tau} = 1-e^{-5} \simeq 1-0.007 = 0.993$ of their final values at the new reference frequency f' before recording/reading out the new *X*, *Y* values.