

Note further that when the {absolute value} of $\text{Re}\{\tilde{\gamma}_{u_k \star p}(\omega)\} \simeq 1$, the k^{th} component of a polyphonic complex sound field $\tilde{S}(\vec{r}, t; \omega)$ is **fully-coherent**, and is one that is associated with **propagating** sound radiation, *e.g.* when a listener's position is far from a point sound source, in the so-called **far-field** region of a sound source, $r \gg \lambda$.

However, when the {absolute value} of $\text{Im}\{\tilde{\gamma}_{u_k \star p}(\omega)\} \simeq 1$ the k^{th} component of a polyphonic complex sound field $\tilde{S}(\vec{r}, t; \omega)$ is {also} **fully-coherent**, but is instead associated with **non-propagating** sound radiation – *i.e.* acoustic energy that is simply sloshing back and forth locally at the listener's point r during each cycle of oscillation, *e.g.* in the so-called **near-field** region of a sound source, $r \ll \lambda$.

Thus, *e.g.* simultaneously exciting the 3 acoustic standing waves associated with the [1,0,0]/[0,1,0]/[0,0,1] axial modes of a cubical 3-D enclosure of side $d = \lambda/2$, with 3-fold degenerate modal frequency $f_{res} \equiv f_{100} = f_{010} = f_{001} = c/2d$, we see that $\text{Re}\{\tilde{\gamma}_{u_k \star p}(\omega_{res})\} \simeq 0$ and $\text{Im}\{\tilde{\gamma}_{u_k \star p}(\omega_{res})\} \simeq 1$ for each of the $k = x, y, z$ components of this complex sound field.

We can additionally define the corresponding $k = x, y, z$ vector components of the **magnitude-squared** version of the **frequency-domain sound field coherence function** $|\tilde{\gamma}_{u \star p}(\omega)|^2$ {*n.b.* a purely **real** quantity}, as:

$$|\tilde{\gamma}_{u \star p}(\omega)|^2 \equiv \tilde{\gamma}_{u \star p}(\omega) \cdot \tilde{\gamma}_{u \star p}^*(\omega) = \frac{|\tilde{S}_{u \star p}(\omega)|^2}{\tilde{S}_{p \star p}(\omega) \cdot \tilde{S}_{u_k \star u_k}(\omega)} = \frac{|\tilde{G}_{u \star p}(\omega)|^2}{\tilde{G}_{p \star p}(\omega) \cdot \tilde{G}_{u_k \star u_k}(\omega)}$$

where:
$$|\tilde{\gamma}_{u \star p}(\omega)|^2 = |\tilde{\gamma}_{u_x \star p}(\omega)|^2 + |\tilde{\gamma}_{u_y \star p}(\omega)|^2 + |\tilde{\gamma}_{u_z \star p}(\omega)|^2$$

The individual $k = x, y, z$ components of the **frequency-domain** the **magnitude-squared** coherence function $|\tilde{\gamma}_{u \star p}(\omega)|^2$ can range from $0 \leq |\tilde{\gamma}_{u_k \star p}(\omega)|^2 \leq 1$. When $|\tilde{\gamma}_{u_k \star p}(\omega)|^2 \simeq 1$, a polyphonic complex sound field $\tilde{S}(\vec{r}, t; \omega)$ is **fully-coherent** (*e.g.* at a listener's position some distance away from a single sound source), whereas when $|\tilde{\gamma}_{u_k \star p}(\omega)|^2 \simeq 0$, the polyphonic complex sound field is **completely incoherent** (*e.g.* at a listener's position deep inside the **reverberant** portion of a polyphonic complex sound field $\tilde{S}(\vec{r}, t; \omega)$ associated with a large listening room and/or auditorium, concert hall, *etc.*).

It can also be seen from the above discussion(s) that the complex 3-D vector coherence function $\tilde{\gamma}_{u \star p}(\omega)$ contains more information (real and imaginary components) and is thus more useful than its purely-real, magnitude-squared version $|\tilde{\gamma}_{u \star p}(\omega)|^2$.