

The $k = x, y, z$ components of the **frequency domain** complex 3-D vector **specific immittances** at the listener's position \vec{r} are:

$$\tilde{z}_{a_k}(\omega) = \frac{\tilde{p}(\omega)}{\tilde{u}_k(\omega)} = \frac{\tilde{u}_k^*(\omega) \cdot \tilde{p}(\omega)}{\tilde{u}_k^*(\omega) \cdot \tilde{u}_k(\omega)} = \frac{\tilde{u}_k^*(\omega) \cdot \tilde{p}(\omega)}{|\tilde{u}_k(\omega)|^2} = \frac{2\tilde{I}_{a_k}(\omega)}{|\tilde{u}_k(\omega)|^2} = \frac{\tilde{S}_{u_k \star p}(\omega)}{\tilde{S}_{u_k \star u_k}(\omega)} = \frac{\tilde{G}_{u_k \star p}(\omega)}{\tilde{G}_{u_k \star u_k}(\omega)} = \frac{\tilde{G}_{p \star u_k}^*(\omega)}{\tilde{G}_{u_k \star u_k}(\omega)}$$

and:
$$\tilde{y}_{a_k}(\omega) = \frac{1}{\tilde{z}_{a_k}(\omega)} = \frac{\tilde{u}_k(\omega)}{\tilde{p}(\omega)} = \frac{\tilde{u}_k(\omega) \cdot \tilde{p}^*(\omega)}{\tilde{p}(\omega) \cdot \tilde{p}^*(\omega)} = \frac{2\tilde{I}_{a_k}(\omega)}{|\tilde{p}(\omega)|^2} = \frac{\tilde{S}_{p \star u_k}(\omega)}{\tilde{S}_{p \star p}(\omega)} = \frac{\tilde{G}_{p \star u_k}(\omega)}{\tilde{G}_{p \star p}(\omega)} = \frac{\tilde{G}_{u_k \star p}^*(\omega)}{\tilde{G}_{p \star p}(\omega)}$$

Since $\tilde{y}_{a_k}(\vec{r}, \omega) = 1/\tilde{z}_{a_k}(\vec{r}, \omega)$, we also see that:

$$\tilde{z}_{a_k}(\omega) = \frac{\tilde{G}_{u_k \star p}(\omega)}{\tilde{G}_{u_k \star u_k}(\omega)} = \frac{\tilde{G}_{p \star u_k}^*(\omega)}{\tilde{G}_{u_k \star u_k}(\omega)} = \frac{1}{\tilde{y}_{a_k}(\omega)} = \frac{\tilde{G}_{p \star p}(\omega)}{\tilde{G}_{p \star u_k}(\omega)} = \frac{\tilde{G}_{p \star p}(\omega)}{\tilde{G}_{u_k \star p}^*(\omega)}$$

Thus, we also see that:

$$\tilde{G}_{p \star p}(\omega) \cdot \tilde{G}_{u_k \star u_k}(\omega) = \tilde{G}_{p \star u_k}(\omega) \cdot \tilde{G}_{u_k \star p}(\omega) = \tilde{G}_{u_k \star p}^*(\omega) \cdot \tilde{G}_{p \star u_k}(\omega) = |\tilde{I}_{a_k}(\omega)|^2$$

We can also define corresponding $k = x, y, z$ vector components of the **frequency-domain** complex 3-D vector **sound field coherence function** $\vec{\tilde{\gamma}}_{p \star u_k}(\omega)$ {*n.b.* essentially the normalized (& dimensionless) complex 3-D vector acoustic intensity} as:

$$\vec{\tilde{\gamma}}_{u_k \star p}(\omega) \equiv \frac{\vec{\tilde{S}}_{u_k \star p}(\omega)}{\sqrt{\tilde{S}_{p \star p}(\omega) \cdot \tilde{S}_{u_k \star u_k}(\omega)}} = \frac{\vec{\tilde{G}}_{u_k \star p}(\omega)}{\sqrt{\tilde{G}_{p \star p}(\omega) \cdot \tilde{G}_{u_k \star u_k}(\omega)}}$$

where:

$$\vec{\tilde{\gamma}}_{u \star p}(\omega) = \tilde{\gamma}_{u_x \star p}(\omega) \hat{x} + \tilde{\gamma}_{u_y \star p}(\omega) \hat{y} + \tilde{\gamma}_{u_z \star p}(\omega) \hat{z}$$

Note that the **magnitudes** of the individual $k = x, y, z$ components of the **frequency-domain** complex 3-D vector **sound field coherence function** are constrained to lie within the range: $0 \leq |\tilde{\gamma}_{u_k \star p}(\omega)| \leq 1$, *i.e.* the individual $k = x, y, z$ vector components are constrained to lie on, or within the **unit circle** in the complex plane, centered at (0,0).

When $|\tilde{\gamma}_{u_k \star p}(\omega)| \simeq 1$, the k^{th} component of a polyphonic complex sound field $\tilde{S}(\vec{r}, t; \omega)$ is **fully-coherent** (*e.g.* at a listener's position r some distance away from a point sound source), whereas when $|\tilde{\gamma}_{u_k \star p}(\omega)| \simeq 0$, the k^{th} component of a polyphonic complex sound field is **completely incoherent** (*e.g.* at a listener's position deep inside the **reverberant** portion of a polyphonic complex sound field $\tilde{S}(\vec{r}, t; \omega)$ associated with a large listening room and/or auditorium, concert hall, *etc.*).