

The **frequency-domain** complex 3-D vector acoustic intensity “amplitude” is:

$$\boxed{\vec{I}_a(\vec{r}, \omega) \equiv \frac{1}{2} \tilde{p}(\vec{r}, \omega) \cdot \vec{u}^*(\vec{r}, \omega)}$$

Which, broken down into its 3 individual space components is:

$$\boxed{\begin{aligned} \tilde{I}_{a_x}(\vec{r}, \omega) \hat{x} &= \frac{1}{2} \tilde{p}(\vec{r}, \omega) \cdot \vec{u}_x^*(\vec{r}, \omega) \hat{x} \\ \tilde{I}_{a_y}(\vec{r}, \omega) \hat{y} &= \frac{1}{2} \tilde{p}(\vec{r}, \omega) \cdot \vec{u}_y^*(\vec{r}, \omega) \hat{y} \\ \tilde{I}_{a_z}(\vec{r}, \omega) \hat{z} &= \frac{1}{2} \tilde{p}(\vec{r}, \omega) \cdot \vec{u}_z^*(\vec{r}, \omega) \hat{z} \end{aligned}}$$

Hence, at the space-point \vec{r} :

$$\boxed{\vec{I}_a(\omega) = \frac{1}{2} \vec{S}_{u \star p}(\omega) = \frac{1}{2} \vec{S}_{p \star u}^*(\omega)} \Rightarrow \boxed{\vec{I}_a(\omega) = \vec{G}_{u \star p}(\omega) = \vec{G}_{p \star u}^*(\omega)} \text{ for } \omega > 0.$$

where:

$$\boxed{\begin{aligned} \vec{I}_a(\omega) &= \vec{G}_{u \star p}(\omega) = \tilde{G}_{u_x \star p}(\omega) \hat{x} + \tilde{G}_{u_y \star p}(\omega) \hat{y} + \tilde{G}_{u_z \star p}(\omega) \hat{z} \\ &= \vec{G}_{p \star u}^*(\omega) = \tilde{G}_{p \star u_x}^*(\omega) \hat{x} + \tilde{G}_{p \star u_y}^*(\omega) \hat{y} + \tilde{G}_{p \star u_z}^*(\omega) \hat{z} \end{aligned}}$$

Note that the complex 3-D vector specific acoustic **impedance** $\vec{z}_a(\vec{r}, \omega)$ and **admittance** $\vec{y}_a(\vec{r}, \omega) = 1/\vec{z}_a(\vec{r}, \omega)$ are **time-independent** quantities. {They are in fact manifestly/intrinsically **frequency domain** quantities!}

Expressed in terms of their **frequency domain** definitions:

$$\boxed{\begin{aligned} \vec{z}_a(\vec{r}, \omega) &\equiv \frac{\tilde{p}(\vec{r}, \omega)}{\vec{u}(\vec{r}, \omega)} = \frac{\tilde{p}(\vec{r}, \omega)}{\tilde{u}_x(\vec{r}, \omega)} \hat{x} + \frac{\tilde{p}(\vec{r}, \omega)}{\tilde{u}_y(\vec{r}, \omega)} \hat{y} + \frac{\tilde{p}(\vec{r}, \omega)}{\tilde{u}_z(\vec{r}, \omega)} \hat{z} \\ &= \tilde{z}_{a_x}(\vec{r}, \omega) \hat{x} + \tilde{z}_{a_y}(\vec{r}, \omega) \hat{y} + \tilde{z}_{a_z}(\vec{r}, \omega) \hat{z} \end{aligned}}$$

and:

$$\boxed{\begin{aligned} \vec{y}_a(\vec{r}, \omega) &\equiv \frac{\vec{u}(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} = \frac{\tilde{u}_x(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} \hat{x} + \frac{\tilde{u}_y(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} \hat{y} + \frac{\tilde{u}_z(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} \hat{z} \\ &= \tilde{y}_{a_x}(\vec{r}, \omega) \hat{x} + \tilde{y}_{a_y}(\vec{r}, \omega) \hat{y} + \tilde{y}_{a_z}(\vec{r}, \omega) \hat{z} \end{aligned}}$$

Note also that:

$$\boxed{\begin{aligned} \vec{z}_a(\vec{r}, \omega) &= \frac{\tilde{p}(\vec{r}, \omega)}{\tilde{u}_x(\vec{r}, \omega)} \cdot \frac{\tilde{u}_x^*(\vec{r}, \omega)}{\tilde{u}_x^*(\vec{r}, \omega)} \hat{x} + \frac{\tilde{p}(\vec{r}, \omega)}{\tilde{u}_y(\vec{r}, \omega)} \cdot \frac{\tilde{u}_y^*(\vec{r}, \omega)}{\tilde{u}_y^*(\vec{r}, \omega)} \hat{y} + \frac{\tilde{p}(\vec{r}, \omega)}{\tilde{u}_z(\vec{r}, \omega)} \cdot \frac{\tilde{u}_z^*(\vec{r}, \omega)}{\tilde{u}_z^*(\vec{r}, \omega)} \hat{z} \\ &= \frac{2\tilde{I}_{a_x}(\vec{r}, \omega)}{|\tilde{u}_x(\vec{r}, \omega)|^2} \hat{x} + \frac{2\tilde{I}_{a_y}(\vec{r}, \omega)}{|\tilde{u}_y(\vec{r}, \omega)|^2} \hat{y} + \frac{2\tilde{I}_{a_z}(\vec{r}, \omega)}{|\tilde{u}_z(\vec{r}, \omega)|^2} \hat{z} \end{aligned}}$$