For practical purposes, it is useful/convenient to redefine the complex ***power*** ***spectral*** ***density*** ***functions*** as ***single-sided*** functions of frequency:

$$\tilde{G}_{x\star y}(\omega) \equiv 2\tilde{S}_{x\star y}(\omega) \text{ for } \omega > 0$$
$$\tilde{G}_{x\star y}(0) \equiv \tilde{S}_{x\star y}(0) \text{ for } \omega = 0$$
$$\tilde{G}_{x\star y}(\omega) \equiv 0 \text{ for } \omega < 0$$

Thus for $\omega > 0$:

$$\tilde{G}_{p\star p}(\omega) \equiv 2\tilde{S}_{p\star p}(\omega) = 2\left|\tilde{p}(\omega)\right|^2 \text{ \{n.b. a purely \underline{real} quantity!\}}$$

$$\tilde{G}_{u\star u}(\omega) \equiv 2\tilde{S}_{u\star u}(\omega) = 2\left|\vec{\tilde{u}}(\omega)\right|^2 = 2\left[\left|\tilde{u}_x(\omega)\right|^2 + \left|\tilde{u}_y(\omega)\right|^2 + \left|\tilde{u}_z(\omega)\right|^2\right]$$
$$= \tilde{G}_{u_x\star u_x}(\omega) + \tilde{G}_{u_y\star u_y}(\omega) + \tilde{G}_{u_z\star u_z}(\omega) \text{ \{n.b. a purely \underline{real} quantity!\}}$$

$$\vec{\tilde{G}}_{p\star u}(\omega) \equiv 2\vec{\tilde{S}}_{p\star u}(\omega) = 2\tilde{p}^*(\omega)\cdot\vec{\tilde{u}}(\omega) \text{ \{n.b. in general, complex\}}$$
$$= \tilde{G}_{p\star u_x}(\omega)\hat{x} + \tilde{G}_{p\star u_y}(\omega)\hat{y} + \tilde{G}_{p\star u_z}(\omega)\hat{z}$$

$$\vec{\tilde{G}}_{u\star p}(\omega) \equiv 2\vec{\tilde{S}}_{u\star p}(\omega) = 2\vec{\tilde{u}}^*(\omega)\cdot\tilde{p}(\omega) = \vec{\tilde{G}}^*_{p\star u}(\omega)$$
$$= \tilde{G}_{u_x\star p}(\omega)\hat{x} + \tilde{G}_{u_y\star p}(\omega)\hat{y} + \tilde{G}_{u_z\star p}(\omega)\hat{z}$$
$$= \tilde{G}^*_{p\star u_x}(\omega)\hat{x} + \tilde{G}^*_{p\star u_y}(\omega)\hat{y} + \tilde{G}^*_{p\star u_z}(\omega)\hat{z}$$

And for $\omega = 0$:

$$\tilde{G}_{p\star p}(0) \equiv \tilde{S}_{p\star p}(0) = \left|\tilde{p}(0)\right|^2 \text{ \{n.b. a purely \underline{real} quantity!\}}$$

$$\tilde{G}_{u\star u}(0) \equiv \tilde{S}_{u\star u}(0) = \left|\vec{\tilde{u}}(0)\right|^2 = 2\left[\left|\tilde{u}_x(0)\right|^2 + \left|\tilde{u}_y(0)\right|^2 + \left|\tilde{u}_z(0)\right|^2\right]$$
$$= \tilde{G}_{u_x\star u_x}(0) + \tilde{G}_{u_y\star u_y}(0) + \tilde{G}_{u_z\star u_z}(0) \text{ \{n.b. in general, complex\}}$$

$$\vec{\tilde{G}}_{p\star u}(0) \equiv \vec{\tilde{S}}_{p\star u}(0) = \tilde{p}^*(0)\cdot\vec{\tilde{u}}(0) \text{ \{n.b. in general, complex\}}$$
$$= \tilde{G}_{p\star u_x}(0)\hat{x} + \tilde{G}_{p\star u_y}(0)\hat{y} + \tilde{G}_{p\star u_z}(0)\hat{z}$$

$$\vec{\tilde{G}}_{u\star p}(0) \equiv \vec{\tilde{S}}_{u\star p}(0) = \vec{\tilde{u}}^*(0)\cdot\tilde{p}(0) = \vec{\tilde{G}}^*_{p\star u}(0) \text{ \{n.b. in general, complex\}}$$
$$= \tilde{G}_{u_x\star p}(0)\hat{x} + \tilde{G}_{u_y\star p}(0)\hat{y} + \tilde{G}_{u_z\star p}(0)\hat{z}$$
$$= \tilde{G}^*_{p\star u_x}(0)\hat{x} + \tilde{G}^*_{p\star u_y}(0)\hat{y} + \tilde{G}^*_{p\star u_z}(0)\hat{z}$$

And for $\omega < 0$: All $\tilde{G}_{x\star y}(\omega) \equiv 0$ for $\omega < 0$, for *x*, *y* = *u*, *p*.