Power Spectral Density Functions:

The ***power spectral density functions*** $\tilde{S}_{p\star p}(\omega)$ and $\tilde{S}_{u\star u}(\omega)$ (*n.b.* complex ***scalar*** quantities) associated with complex scalar pressure $\tilde{p}(\vec{r},t)$ and vector particle velocity $\vec{\tilde{u}}(\vec{r},t)$ are respectively:

$$\boxed{\tilde{S}_{p\star p}(\omega)=\int_{-\infty}^{+\infty}\tilde{h}_{p\star p}(t)e^{-i\omega t}dt=\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\tilde{p}^*(\tau)\cdot\tilde{p}(t+\tau)d\tau\right]e^{-i\omega t}dt=\tilde{p}^*(\omega)\cdot\tilde{p}(\omega)=\left|\tilde{p}(\omega)\right|^2}$$

and: $$\boxed{\tilde{S}_{u\star u}(\omega)=\int_{-\infty}^{+\infty}\tilde{h}_{u\star u}(t)e^{-i\omega t}dt=\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\vec{\tilde{u}}^*(\tau)\bullet\vec{\tilde{u}}(t+\tau)d\tau\right]e^{-i\omega t}dt=\vec{\tilde{u}}^*(\omega)\bullet\vec{\tilde{u}}(\omega)=\left|\vec{\tilde{u}}(\omega)\right|^2}$$

Explicitly writing out the latter relation in terms of its *x*, *y*, and *z*-components:

$$\boxed{\tilde{S}_{u\star u}(\omega)=\left|\vec{\tilde{u}}(\omega)\right|^2}\Rightarrow\boxed{\tilde{S}_{u_x\star u_x}(\omega)+\tilde{S}_{u_y\star u_y}(\omega)+\tilde{S}_{u_z\star u_z}(\omega)=\left|\tilde{u}_x(\omega)\right|^2+\left|\tilde{u}_y(\omega)\right|^2+\left|\tilde{u}_z(\omega)\right|^2}$$

The dimensionful physical units of $\tilde{S}_{p\star p}(f)$ and $\tilde{S}_{u\star u}(f)$ are Pa^2/Hz and $(m/s)^2/Hz$, respectively.
The dimensionful physical units of $\tilde{S}_{p\star p}(\omega)$ and $\tilde{S}_{u\star u}(\omega)$ are Pa^2-*s*/*rad* and $(m/s)^2$-*s*/*rad*, respectively.

We also see that the 3-D vector ***power spectral density functions*** $\vec{\tilde{S}}_{p\star u}(\omega)$ and $\vec{\tilde{S}}_{u\star p}(\omega)$ – related to the ***frequency-domain*** complex 3-D vector acoustic intensity $\vec{\tilde{I}}_a(\omega)=\frac{1}{2}p(\omega)\cdot\vec{\tilde{u}}^*(\omega)$ are:

$$\boxed{\vec{\tilde{S}}_{p\star u}(\omega)=\int_{-\infty}^{+\infty}\vec{\tilde{h}}_{p\star u}(t)e^{-i\omega t}dt=\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\tilde{p}^*(\tau)\cdot\vec{\tilde{u}}(t+\tau)d\tau\right]e^{-i\omega t}dt=\tilde{p}^*(\omega)\cdot\vec{\tilde{u}}(\omega)}$$

and: $$\boxed{\vec{\tilde{S}}_{u\star p}(\omega)=\int_{-\infty}^{+\infty}\vec{\tilde{h}}_{u\star p}(t)e^{-i\omega t}dt=\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\vec{\tilde{u}}^*(\tau)\cdot\tilde{p}(t+\tau)d\tau\right]e^{-i\omega t}dt=\vec{\tilde{u}}^*(\omega)\cdot\tilde{p}(\omega)}$$

Note that: $$\boxed{\left\{\vec{\tilde{S}}_{u\star p}(\omega)=\vec{\tilde{u}}^*(\omega)\cdot\tilde{p}(\omega)\right\}=\left\{\vec{\tilde{S}}_{p\star u}(\omega)=\tilde{p}^*(\omega)\cdot\vec{\tilde{u}}(\omega)\right\}^*=\left\{\vec{\tilde{S}}^*_{p\star u}(\omega)=\tilde{p}(\omega)\cdot\vec{\tilde{u}}^*(\omega)\right\}}.$$

Note also that for *x*,*y* = *u*,*p*: $\boxed{\mathrm{Re}\left\{\tilde{S}_{x\star y}(\omega)\right\}=\mathrm{Re}\left\{\tilde{S}_{x\star y}(-\omega)\right\}}$ whereas: $\boxed{\mathrm{Im}\left\{\tilde{S}_{x\star y}(\omega)\right\}=-\mathrm{Im}\left\{\vec{\tilde{S}}_{x\star y}(-\omega)\right\}}$.

The dimensionful physical units of $\tilde{S}_{p\star u}(f)$ and $\tilde{S}_{u\star p}(f)$ are *Watts/m*^{2}*/Hz*.
The dimensionful physical units of $\tilde{S}_{p\star u}(\omega)$ and $\tilde{S}_{u\star p}(\omega)$ are *Watt-s/m*^{2}*/rad*.

Generically, the ***power spectral density functions*** $\tilde{S}_{x\star y}(\omega)$ are defined for ***all*** positive ***and*** negative frequencies, and as such, each $\tilde{S}_{x\star y}(\omega)$ can be represented by ***pairs*** of ***counter-rotating phasors*** in the ***complex plane***.