Cross-Correlation:

The ***cross-correlation*** of a complex ***time domain*** function $\tilde{f}(t)$ with another $\tilde{g}(t)$ is a specialized type of ***convolution***, involving complex conjugation:

$$\boxed{\tilde{h}_{f\star g}(t) \equiv \tilde{f}(t)\star\tilde{g}(t) = \tilde{f}^*(-t)\otimes\tilde{g}(t) = \int_{-\infty}^{+\infty}\tilde{f}^*(-\tau)\cdot\tilde{g}(t-\tau)\,d\tau = \int_{-\infty}^{+\infty}\tilde{f}^*(\tau)\cdot\tilde{g}(t+\tau)\,d\tau}$$

The $\star$ symbol denotes the ***cross-correlation*** operation. *n.b.* $\tilde{h}_{f\star g}(t)$ has units of $\tilde{f}(t)\cdot\tilde{g}(t)\cdot sec$.

The Fourier transform of the ***cross-correlation*** relation is: $\tilde{h}_{f\star g}(\omega) = \mathcal{F}\left\{\tilde{h}_{f\star g}(t)\right\} \equiv \int_{-\infty}^{+\infty}\tilde{h}_{f\star g}(t)e^{-i\omega t}dt$, the ***frequency domain*** representation of complex ***time domain cross-correlation***. It can be shown that:

$$\boxed{\tilde{h}_{f\star g}(\omega) = \mathcal{F}\left\{\tilde{h}_{f\star g}(t)\right\} = \mathcal{F}\left\{\tilde{f}^*(-t)\otimes\tilde{g}(t)\right\} = \mathcal{F}\left\{\tilde{f}^*(-t)\right\}\cdot\mathcal{F}\left\{\tilde{g}(t)\right\} = \tilde{f}^*(\omega)\cdot\tilde{g}(\omega)}.$$

The $\cdot$ symbol denotes simple ***multiplication***. *n.b.* $\tilde{h}_{f\star g}(\omega)$ has physical units of $\tilde{f}^*(\omega)\cdot\tilde{g}(\omega)$.

Auto-Correlation (*aka* Self-Correlation):

Note that the ***auto-correlation*** of a complex time-domain function $\tilde{f}(t)$ with ***itself*** is simply a specialized type of ***cross-correlation***, also involving complex conjugation:

$$\boxed{\tilde{h}_{f\star f}(t) \equiv \tilde{f}(t)\star\tilde{f}(t) = \tilde{f}^*(-t)\otimes\tilde{f}(t) = \int_{-\infty}^{+\infty}\tilde{f}^*(-\tau)\cdot\tilde{f}(t-\tau)\,d\tau = \int_{-\infty}^{+\infty}\tilde{f}^*(\tau)\cdot\tilde{f}(t+\tau)\,d\tau}$$

The Fourier transform of the ***auto-correlation*** relation is: $\tilde{h}_{f\star f}(\omega) = \mathcal{F}\left\{\tilde{h}_{f\star f}(t)\right\} \equiv \int_{-\infty}^{+\infty}\tilde{h}_{f\star f}(t)e^{-i\omega t}dt$, the ***frequency domain*** representation of complex ***time domain auto-correlation***. It can be shown that:

$$\boxed{\tilde{h}_{f\star f}(\omega) = \mathcal{F}\left\{\tilde{h}_{f\star f}(t)\right\} = \mathcal{F}\left\{\tilde{f}(t)\star\tilde{f}(t)\right\} = \mathcal{F}\left\{\tilde{f}^*(-t)\otimes\tilde{f}(t)\right\} = \mathcal{F}\left\{\tilde{f}^*(-t)\right\}\cdot\mathcal{F}\left\{\tilde{f}(t)\right\} = \tilde{f}^*(\omega)\cdot\tilde{f}(\omega)}$$

Note that $\tilde{h}_{f\star f}(t)$ has physical units of $\tilde{f}(t)\cdot\tilde{f}(t)\cdot sec$; $\tilde{h}_{f\star f}(\omega)$ has physical units of $\tilde{f}^*(\omega)\cdot\tilde{f}(\omega)$.

The Weiner-Khintchine Theorem:

The ***Weiner-Khintchine Theorem*** relates the ***time domain auto-correlation*** function $\tilde{h}_{f\star f}(t)$ to the ***frequency domain power spectral density*** function $\tilde{S}_{f\star f}(\omega)$ (and vice versa) via the following Fourier transforms:

$$\boxed{\tilde{S}_{f\star f}(\omega) = \int_{-\infty}^{+\infty}\tilde{h}_{f\star f}(t)e^{-i\omega t}dt} \quad \text{and:} \quad \boxed{\tilde{h}_{f\star f}(t) = \tfrac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{S}_{f\star f}(\omega)e^{+i\omega t}d\omega}.$$

These results can be generalized to:

$$\boxed{\tilde{S}_{f\star g}(\omega) = \int_{-\infty}^{+\infty}\tilde{h}_{f\star g}(t)e^{-i\omega t}dt} \quad \text{and:} \quad \boxed{\tilde{h}_{f\star g}(t) = \tfrac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{S}_{f\star g}(\omega)e^{+i\omega t}d\omega}.$$