sec, *meters*, *etc*. However, since frequencies (f) are usually expressed in *Hz* (= cycles per second = *sec*$^{-1}$), the dimensionful physical units of ***frequency domain*** $\tilde{g}(f)$ are more commonly expressed as *RMS Volts*/*Hz*, *Pascals*/*Hz*, (*meters*/*sec*)/*Hz*, *etc*., respectively.

The ***angular frequency*** Fourier transform of $\tilde{g}(t)$, $\tilde{g}(\omega) = \mathcal{F}\{\tilde{g}(t)\} \equiv \int_{-\infty}^{+\infty} \tilde{g}(t) e^{-i\omega t} dt$ have dimensionful physical units of *RMS Volt*-sec/*rad*, *Pascal*-*sec*/*rad*, *meters*/*rad*, *etc*., respectively.

If we now explicitly insert $\tilde{g}(t) = p_o e^{+i\omega_o t}$ into the expression for the Fourier transform of the ***time domain*** $\tilde{g}(t)$ to the ***frequency domain***:

$$\tilde{g}(\omega) = \mathcal{F}\{\tilde{g}(t)\} \equiv \int_{-\infty}^{+\infty} \tilde{g}(t) e^{-i\omega t} dt = \int_{-\infty}^{+\infty} p_o e^{+i\omega_o t} e^{-i\omega t} dt = p_o \underbrace{\int_{-\infty}^{+\infty} e^{+i(\omega_o - \omega)t} dt}_{=2\pi\delta(\omega_o - \omega)} = p_o \cdot 2\pi\delta(\omega_o - \omega)$$

where the delta function $\delta(\omega_o - \omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i(\omega_o - \omega)t} dt$, has the dimensionful physical units of the ***inverse*** of its argument, *i.e*. $1/\omega = 1/2\pi f$ (= seconds/radian, or equivalently, 1/*radian*-*Hz*).

We insert the ***frequency domain*** $\tilde{g}(\omega) = p_o \cdot 2\pi\delta(\omega_o - \omega)$ into the expression for the ***inverse*** Fourier transform to obtain the ***time domain*** representation $\tilde{g}(t)$:

$$\tilde{g}(t) = \mathcal{F}^{-1}\{\tilde{g}(\omega)\} \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{g}(\omega) e^{+i\omega t} d\omega = \frac{1}{\cancel{2\pi}}\int_{-\infty}^{+\infty} p_o \cdot \cancel{2\pi}\delta(\omega_o - \omega) e^{+i\omega t} d\omega = p_o e^{+i\omega_o t}$$

where we have used the relations: $\int_{-\infty}^{+\infty} \delta(x_o - x) dx = 1$ and: $\int_{-\infty}^{+\infty} g(x) \cdot \delta(x_o - x) dx = g(x_o)$.

Thus, we see that an infinitely long/continuous complex exponential ***time domain*** signal $\tilde{g}(t) = p_o e^{+i\omega_o t}$ corresponds to an infinitely sharp/narrow "spike" in the ***frequency domain*** $\tilde{g}(\omega) = p_o \cdot 2\pi\delta(\omega_o - \omega)$, as shown graphically in the two figures below: