Spectral Analysis Approach to Measurements of Complex Sound Fields

A complex ***polyphonic*** sound field may consist of ***many*** individual sound fields, each one with its own characteristic frequency {and/or frequencies}, which may {or may not} necessarily be related in phase to other polyphonic components of the overall sound field. The individual components of the overall complex polyphonic sound field may also emanate from their own sound source(s) at different spatial locations. In such situations, measurements of the so-called ***frequency domain power spectral densities*** $\tilde{S}(\vec{r},t;\omega)$ associated with complex overpressure $\tilde{p}(\vec{r},t)$ and particle velocity $\tilde{\vec{u}}(\vec{r},t)$ are carried out in order to determine the nature of the overall sound field at the listener's position $\vec{r}$.

We use Fourier transforms to obtain the ***frequency domain*** complex overpressure $\tilde{p}(\vec{r},\omega)$ and particle velocity $\tilde{\vec{u}}(\vec{r},\omega)$ associated with their ***time domain*** complex overpressure $\tilde{p}(\vec{r},t)$ and particle velocity $\tilde{\vec{u}}(\vec{r},t)$ counterparts.

For any ***continuous***, mathematically ***well-behaved*** complex ***time domain*** function $\tilde{g}(t)$, the Fourier transform of ***time domain*** $\tilde{g}(t)$ to the ***frequency domain*** f (and/or ***angular frequency domain*** $\omega = 2\pi f$) is:

$$\boxed{\tilde{g}(f) = \tilde{g}(\omega) = \mathcal{F}\{\tilde{g}(t)\} \equiv \int_{-\infty}^{+\infty} \tilde{g}(t) e^{-i\omega t} dt = \int_{-\infty}^{+\infty} \tilde{g}(t) e^{-i2\pi f t} dt}$$

The ***inverse*** Fourier transform(s) of ***frequency domain*** $\tilde{g}(f) = \tilde{g}(\omega)$ to the ***time domain*** is:

$$\boxed{\tilde{g}(t) = \mathcal{F}^{-1}\{\tilde{g}(\omega)\} \equiv \tfrac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{g}(\omega) e^{+i\omega t} d\omega = \int_{-\infty}^{+\infty} \tilde{g}(f) e^{+i2\pi f t} df} \quad \text{since: } \boxed{d\omega = 2\pi \cdot df}$$

Note that the choice of the $\pm$ signs in the complex exponential factors in the above Fourier transform expressions is ***not*** arbitrary – long ago, we specified the $e^{+i\omega t}$ convention for use in our physically-measureable quantities, *e.g.* $\tilde{p}(t) = p_o e^{+i\omega t}\ (Pascals)$, $\tilde{u}_{\|}(t) = u_{\|}^{o} e^{+i\omega t}\ (m/sec^{-1})$, *etc.*

A Simple Example of the Use of Fourier Transforms:

Suppose we have a pure-tone/single frequency $(\omega \equiv \omega_o = 2\pi f_o)$ complex ***time domain*** signal $\tilde{g}(t) = p_o e^{+i\omega_o t}\ (Pascals)$, where, for simplicity's sake {***here***}, the overpressure amplitude $p_o\ (Pascals)$ is a purely real number (*i.e.* a constant). Depending on the physical quantity that the ***time domain*** signal $\tilde{g}(t)$ actually represents, the ***time domain*** signal $\tilde{g}(t)$ has dimensionful physical units associated with it – *e.g. RMS Volts*, *Pascals*, *meters/second*, *etc*.

Hence, note that the physical units associated with the ***frequency domain*** Fourier transform of $\tilde{g}(t)$, $\tilde{g}(f) = \mathcal{F}\{\tilde{g}(t)\} \equiv \int_{-\infty}^{+\infty} \tilde{g}(t) e^{-i2\pi f t} dt$ will correspondingly be *RMS Volt-sec*, *Pascal-*