A ***dual-channel*** *LIA* uses ***two*** so-called **Phase-Sensitive Detectors** (*PSD's*) to:

(*a*.) Carry out the mathematical operation of ***multiplication*** of the ***output*** signal with the ***reference/input*** sine-wave signal, and also with a +90° ***phase-shifted copy*** of the ***reference/input*** sine-wave signal, and then:

(*c*.) The two *PSD* ***product*** signals are then either ***time-averaged*** or ***low-pass filtered*** to obtain quasi-*DC* voltages that are representative of the ***in-phase*** and ***90°-out-of-phase/quadrature*** amplitude components of the harmonic (*i.e.* periodic) output response signal, respectively.

The *LIA's* ***reference*** sine-wave signal is:

$$V^{Ref}(\omega,t) = V_o^{Ref}\cos(\omega t + \varphi_R)$$

The +90° ***phase-shifted copy*** of the ***reference*** sine-wave signal is:

$$\begin{aligned} V_{\pi/2}^{Ref}(\omega,t) &= V_o^{Ref}\cos(\omega t + \varphi_R + \pi/2) \\ &= V_o^{Ref}\left[\cos(\omega t + \varphi_R)\cancel{\cos(\pi/2)} - \sin(\omega t + \varphi_R)\sin(\pi/2)\right] \\ &= -V_o^{Ref}\sin(\omega t + \varphi_R) \end{aligned}$$

The generic "black-box" system's output response signal (input to the dual-channel *LIA*) is:

$$V^{Sig}(\omega,t) = V_o^{Sig}(\omega)\cos(\omega t + \varphi_S(\omega))$$

The signal ***multiplication*** operation carried out by the 1st Phase-Sensitive Detector (= PSD_X) is:

$$V^{PSD_X}(\omega,t) \equiv V^{Sig}(\omega,t)\otimes V^{Ref}(\omega,t) = V_o^{Sig}(\omega)\cos(\omega t + \varphi_S(\omega))\otimes V_o^{Ref}\cos(\omega t + \varphi_R)$$

The signal ***multiplication*** operation carried out by the 2nd Phase-Sensitive Detector (= PSD_Y) is:

$$V^{PSD_Y}(\omega,t) \equiv V^{Sig}(\omega,t)\otimes V_{\pi/2}^{Ref}(\omega,t) = -V_o^{Sig}(\omega)\cos(\omega t + \varphi_S(\omega))\otimes V_o^{Ref}\sin(\omega t + \varphi_R)$$

We define: $x \equiv \omega t + \varphi_S(\omega)$ and: $y \equiv \omega t + \varphi_R$. Then using Euler's formulas: $\cos x = \tfrac{1}{2}\left(e^{ix} + e^{-ix}\right)$ and: $\sin x = \tfrac{1}{2i}\left(e^{ix} - e^{-ix}\right)$, the two *PSD* product terms can be rewritten as:

$$\begin{aligned} \cos x\cos y &= \tfrac{1}{4}\left(e^{ix}+e^{-ix}\right)\left(e^{iy}+e^{-iy}\right) = \tfrac{1}{4}\left(e^{i(x+y)} + e^{-i(x+y)} + e^{i(x-y)} + e^{-i(x-y)}\right) \\ &= \tfrac{1}{4}\left(2\cos(x+y) + 2\cos(x-y)\right) = \tfrac{1}{2}\left(\cos(x+y) + \cos(x-y)\right) \end{aligned}$$

$$\begin{aligned} \cos x\sin y &= \tfrac{1}{4i}\left(e^{ix}+e^{-ix}\right)\left(e^{iy}-e^{-iy}\right) = \tfrac{1}{4i}\left(e^{i(x+y)} - e^{-i(x+y)} - e^{i(x-y)} + e^{-i(x-y)}\right) \\ &= \tfrac{1}{4}\left(2\sin(x+y) - 2\sin(x-y)\right) = \tfrac{1}{2}\left(\sin(x+y) - \sin(x-y)\right) \end{aligned}$$