Measurement of Complex Sound Fields – Part 2:

The Use of Lock-In Amplifiers for Phase-Sensitive Measurements of Complex Harmonic Sound Fields

What do Lock-In Amplifiers (*LIA′s*) do, and how do they work?

Consider a "generic" experimental situation where a harmonic (*i.e*. periodic/pure-tone/single-frequency) signal of {angular} frequency $\omega = 2\pi f$ is used as a "stimulus" to excite a system (*i.e*. input a known signal into an unknown "black-box"). We are interested in measuring the ***linear***, but possibly ***complex*** response of the "black-box" system to the input stimulus signal – *i.e*. its "output" signal strength (amplitude) and phase of the output signal ***relative*** to the input "stimulus" signal (*aka* the input ***reference*** signal). This "generic" situation is shown in the figure below:

Input Stimulus: $V^{Ref}(\omega,t) = V_o^{Ref}\cos(\omega t + \varphi_R)$ → Generic "Black-Box" System → Output Response: $V^{Sig}(\omega,t) = V_o^{Sig}(\omega)\cos(\omega t + \varphi_S(\omega))$

A ***dual-channel*** *LIA* is a narrow-bandwidth electronic device that measures/determines the ***in-phase*** and ***90°-out-of-phase/quadrature amplitude*** components of the response signal output from a generic "black-box" system ***relative*** to a harmonic/pure-tone/single-frequency reference signal input to that system. The generic "black-box" system's output response signal is:

$$V^{Sig}(\omega,t) \equiv V_o^{Sig}(\omega)\cos(\omega t + \varphi_S(\omega)) \text{ of \{angular\} frequency } \omega = 2\pi f\,.$$

Note that in general, both the system's output signal ***amplitude*** $V_o^{Sig}(\omega)$ and ***phase*** $\varphi_S(\omega)$ are frequency-dependent quantities. Whether they in fact are (or are not) depends on the detailed physics associated with how the system's output signal is actually produced, *i.e*. what the output response signal $V^{Sig}(\omega,t)$ physically represents.

Note that the ***in-phase*** and ***90°-out-of-phase/quadrature*** components of the harmonic output response signal ***amplitude*** $V^{Sig}(\omega,t)$ are defined ***relative*** to a ***reference/input*** sine-wave of the same frequency *f*:

$$V^{Ref}(\omega,t) \equiv V_o^{Ref}\cos(\omega t + \varphi_R) \text{ of \{angular\} frequency } \omega = 2\pi f$$

Note further that {here} the ***reference/input*** signal's ***amplitude*** V_o^{Ref} and {absolute} ***phase*** φ_R are both ***constants***, *i.e*. ***time-independent***.