Transducers Used for the Measurement of Particle Velocity:

How can one measure particle velocity $\vec{\tilde{u}}(\vec{r},t)$? There are two ways – one is to build a device which is ***directly*** sensitive to particle velocity, the other is to exploit what the {linearized} Euler equation for inviscid fluid flow tells us:

$$\frac{\partial \vec{\tilde{u}}(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla}\tilde{p}(\vec{r},t)$$

If we focus, for the moment on ***one*** spatial component of this equation, *e.g.* in the $\hat{z}$ -direction, this equation in Cartesian coordinates becomes simply:

$$\frac{\partial \tilde{u}_z(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\frac{\partial \tilde{p}(\vec{r},t)}{\partial z} \Rightarrow -\frac{1}{\rho_o}\frac{\Delta_z \tilde{p}(\vec{r},t)}{\Delta z}$$

Euler's equation tells us that if we measure the pressure ***gradient*** {an over-pressure ***difference***} $\Delta_z \tilde{p}(\vec{r},t) \equiv \tilde{p}(z_2,t) - \tilde{p}(z_1,t)$ over a ***small*** separation distance $\Delta z \equiv z_2 - z_1$ (*n.b.* with $\Delta z \ll \lambda$), then Euler's equation tells us that the pressure gradient is proportional to the time rate of change of the *z*-component of the particle velocity $\partial \tilde{u}_z(\vec{r},t)/\partial t$! Thus, if we integrate the pressure gradient $\Delta_z \tilde{p}(\vec{r},t)/\Delta z$ with respect to time, the integral is ***proportional*** to the *z*-component of the particle velocity:

$$\tilde{u}_z(\vec{r},t) = -\frac{1}{\rho_o \Delta z}\int_{t'=-\infty}^{t'=t} \Delta_z \tilde{p}(\vec{r},t')\,dt' = -\frac{1}{\rho_o(z_2 - z_1)}\int_{t'=-\infty}^{t'=t} \left[\tilde{p}(z_2,t') - \tilde{p}(z_1,t')\right]dt'$$

Likewise, if we then also measure the instantaneous pressure gradients $\Delta_x \tilde{p}(\vec{r},t)$ $\left(\Delta_y \tilde{p}(\vec{r},t)\right)$ over small distances Δx (Δy) in the $\hat{x}$ $(\hat{y})$-directions, respectively, and then integrate these two signals, then we also obtain measurements of:

$$\tilde{u}_x(\vec{r},t) = -\frac{1}{\rho_o \Delta x}\int_{t'=-\infty}^{t'=t} \Delta_x \tilde{p}(\vec{r},t')\,dt' = -\frac{1}{\rho_o(x_2 - x_1)}\int_{t'=-\infty}^{t'=t} \left[\tilde{p}(x_2,t') - \tilde{p}(x_1,t')\right]dt'$$

and:
$$\tilde{u}_y(\vec{r},t) = -\frac{1}{\rho_o \Delta y}\int_{t'=-\infty}^{t'=t} \Delta_y \tilde{p}(\vec{r},t')\,dt' = -\frac{1}{\rho_o(y_2 - y_1)}\int_{t'=-\infty}^{t'=t} \left[\tilde{p}(y_2,t') - \tilde{p}(y_1,t')\right]dt'$$

To measure a pressure gradient $\Delta_r \tilde{p}(\vec{r},t) = \tilde{p}(\vec{r}_2,t) - \tilde{p}(\vec{r}_1,t)$ between two points one can use *e.g.* two {hopefully} identical/perfectly frequency/phase-matched microphones (with $S_{mic1} = S_{mic2}$), and then use (or build) a (precision) low-noise ***differential*** preamp to take the voltage difference $\Delta\tilde{V}(t) = \tilde{V}_{mic2}(\vec{r}_2,t) - \tilde{V}_{mic1}(\vec{r}_1,t)$ between the two signals, since:

$$\Delta_r \tilde{p}(\vec{r},t) = \tilde{p}(\vec{r}_2,t) - \tilde{p}(\vec{r}_1,t) = \frac{\tilde{V}_{mic2}(\vec{r}_2,t)}{S_{mic2}} - \frac{\tilde{V}_{mic1}(\vec{r}_1,t)}{S_{mic1}}$$