

The **phase** of the complex displacement amplitude $\tilde{x}_o(\omega)$ of the microphone diaphragm is:

$$\varphi_x(\omega) \equiv \tan^{-1} \left(\frac{\text{Im}\{\tilde{x}_o(\omega)\}}{\text{Re}\{\tilde{x}_o(\omega)\}} \right) = \tan^{-1} \left(\frac{-\omega\gamma_{dia}}{(\omega_o^2 - \omega^2)} \right) = -\tan^{-1} \left(\frac{\omega\gamma_{dia}}{(\omega_o^2 - \omega^2)} \right)$$

The **magnitude** of the complex displacement amplitude $\tilde{x}_o(\omega)$ of the microphone diaphragm is:

$$|\tilde{x}_o(\omega)| \equiv \sqrt{x_{or}^2(\omega) + x_{oi}^2(\omega)} = \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2 \gamma_{dia}^2}}{(\omega_o^2 - \omega^2)^2 + \omega^2 \gamma_{dia}^2} = \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{1}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2 \gamma_{dia}^2}}$$

In order for a microphone to be effective as a pressure transducer, the face of the diaphragm of the microphone must (a.) be nearly massless – *i.e.* $m_{dia} \sim 0$ and (b.) have great stiffness, so that the diaphragm acts like a piston. On the other hand, the supporting edge of the diaphragm, where it is attached to the rigid structural body of the microphone must be reasonably compliant, in order to allow the back-and-forth motion of the diaphragm along its \hat{n} -axis *e.g.* in response to a harmonic over-pressure amplitude. Let us see what is required in terms of damping the motion of the diaphragm.

If we define $x(\omega) \equiv \omega/\omega_o$ and $y \equiv \gamma_{dia}/\omega_o$ then we can rewrite the {normalized} magnitude of the complex displacement amplitude $\tilde{x}_o(\omega)$ and phase as:

$$\text{Res}(x(\omega)) \equiv \frac{\omega_o^2 |\tilde{x}_o(\omega)|}{\left(\frac{p_o A_{dia}}{m_{dia}} \right)} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_o} \right)^2 \right]^2 + \left(\frac{\omega}{\omega_o} \right)^2 \left(\frac{\gamma_{dia}}{\omega_o} \right)^2}} = \frac{1}{\sqrt{(1-x^2)^2 + x^2 y^2}}$$

$$\varphi_x(\omega) = -\tan^{-1} \left(\frac{\omega\gamma_{dia}}{(\omega_o^2 - \omega^2)} \right) = -\tan^{-1} \left(\frac{\left(\frac{\omega}{\omega_o} \right) \left(\frac{\gamma_{dia}}{\omega_o} \right)}{1 - \left(\frac{\omega}{\omega_o} \right)^2} \right) = -\tan^{-1} \left(\frac{x \cdot y}{1-x^2} \right)$$

We have the usual three conditions of **underdamped**, **critically damped** and **overdamped** motion for $y \equiv \gamma_{dia}/\omega_o \ll 1$, $y \equiv \gamma_{dia}/\omega_o = 1$ and $y \equiv \gamma_{dia}/\omega_o \gg 1$, respectively. The following two color-coded figures show normalized $\text{Res}(x)$ vs. x and $\text{Phase}(x)$ vs. x for $y = 0.05$ (pink), 1.0 (cyan) and 10.0 (violet):