

Since all forces act along/against the same direction (*i.e.* the \hat{n} -axis = the outward pointing normal to the plane of the microphone diaphragm), then using the relations:

$$\vec{v}_{dia}(t) = \partial \vec{x}_{dia}(t) / \partial t \equiv \dot{\vec{x}}_{dia}(t) \quad \text{and:} \quad \vec{a}_{dia}(t) = \partial \vec{v}_{dia}(t) / \partial t = \partial^2 \vec{x}_{dia}(t) / \partial t^2 \equiv \ddot{\vec{x}}_{dia}(t),$$

and dropping the vector arrows, we arrive at the 2nd order **inhomogeneous** differential equation of 1-D motion of the microphone diaphragm:

$$m_{dia} \ddot{x}_{dia}(t) + m_{dia} \gamma_{dia} \dot{x}_{dia}(t) + k_{dia} x_{dia}(t) = \tilde{p}(t) A_{dia}$$

The general solution to this equation for steady-state driven harmonic motion of the diaphragm is of the form $\tilde{x}_{dia}(t) = \tilde{x}_o(\omega) e^{i\omega t}$. For un-damped, non-driven (*i.e.* $\gamma_{dia} = 0$ and $\tilde{p}(t) = 0$) steady-state motion of the diaphragm, the natural resonant angular frequency is $\omega_o \equiv \sqrt{k_{dia}/m_{dia}}$. For damped and driven steady-state harmonic motion (*i.e.* $\gamma_{dia} \neq 0$ and $\tilde{p}(t) = p_o e^{i\omega t} \neq 0$):

$$-\omega^2 m_{dia} \tilde{x}_o(\omega) e^{i\omega t} + i\omega m_{dia} \gamma_{dia} \tilde{x}_o(\omega) e^{i\omega t} + k_{dia} \tilde{x}_o(\omega) e^{i\omega t} = p_o e^{i\omega t} A_{dia}$$

or:
$$\left[-\omega^2 m_{dia} + i\omega m_{dia} \gamma_{dia} + k_{dia} \right] \tilde{x}_o(\omega) = p_o A_{dia}$$

Dividing though by m_{dia} :
$$\left[-\omega^2 + i\omega \gamma_{dia} + (k_{dia}/m_{dia}) \right] \tilde{x}_o(\omega) = p_o A_{dia} / m_{dia}$$

Using the natural angular resonant frequency relation $\omega_o^2 \equiv (k_{dia}/m_{dia})$:

$$\left[-\omega^2 + i\omega \gamma_{dia} + \omega_o^2 \right] \tilde{x}_o(\omega) = p_o A_{dia} / m_{dia}$$

or:

$$\begin{aligned} \tilde{x}_o(\omega) &= \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{1}{\left[(\omega_o^2 - \omega^2) + i\omega \gamma_{dia} \right]} = \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{1}{\left[(\omega_o^2 - \omega^2) + i\omega \gamma_{dia} \right]} \cdot \frac{\left[(\omega_o^2 - \omega^2) - i\omega \gamma_{dia} \right]}{\left[(\omega_o^2 - \omega^2) - i\omega \gamma_{dia} \right]} \\ &= \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{\left[(\omega_o^2 - \omega^2) - i\omega \gamma_{dia} \right]}{(\omega_o^2 - \omega^2)^2 + \omega^2 \gamma_{dia}^2} \end{aligned}$$

The **real/in-phase component** of the complex displacement amplitude $\tilde{x}_o(\omega)$ of the microphone diaphragm is:

$$x_{or}(\omega) \equiv \text{Re} \{ \tilde{x}_o(\omega) \} = \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2)^2 + \omega^2 \gamma_{dia}^2}$$

The **imaginary/quadrature component** of the complex displacement amplitude $\tilde{x}_o(\omega)$ of the microphone diaphragm is:

$$x_{oi}(\omega) \equiv \text{Im} \{ \tilde{x}_o(\omega) \} = - \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{\omega \gamma_{dia}}{(\omega_o^2 - \omega^2)^2 + \omega^2 \gamma_{dia}^2}$$