<u>Example # 4: The *Physical* "Point" Monopole Sound Source</u>:

At very low frequencies, a loudspeaker mounted in a fully-enclosed/sealed cabinet of characteristic dimension *a*, with $ka \ll 1$ (*i.e.* $f \ll c/2\pi a$ {using $k \equiv 2\pi/\lambda$, $\omega \equiv 2\pi f$ and $c = f\lambda = \omega/k$ }) approximates a "point" monopole sound source – the directivity factor, *Q* of a typical enclosed loudspeaker is very nearly 1 (*i.e.* isotropic) at frequencies $f \ll c/2\pi a$. For example, for a typical "bookshelf"-type loudspeaker with $a \sim 1\ ft \sim 0.3\ m$, then $f \ll c/2\pi a = 343/0.6\pi \sim 180\ Hz$, or equivalently $\lambda \gg 1.9\ m$.

If we use the "***spherical cow***" approximation, *i.e.* model a physical monopole sound source as a radially-pulsating sphere of radius *a*, subject to the restriction $ka \ll 1$, then the acoustical properties of such a device (for *r* > *a*) will closely approximate that of an ideal, point monopole sound source.

How do we characterize the ***<u>strength</u>*** of a ***<u>physical</u>*** monopole sound source – *i.e.* a radially-pulsating sphere of radius *a*? Typically, this is done by considering the {complex} ***<u>volumetric velocity</u>*** (*aka* ***<u>volume</u>*** ***<u>velocity</u>***) of the physical monopole source, evaluated at the radius *a* of the sphere – the {radial} outward volume rate (or flow) of fluid (*i.e.* air) from this sphere:

$$\tilde{Q}_a \cdot e^{i\omega t} = \int_S \tilde{u}_r(r=a,t)\,\hat{r}\bullet\hat{n}dS = 4\pi a \frac{\tilde{B}}{z_o}\left[1-\frac{i}{ka}\right]e^{-ika}\cdot e^{i\omega t} \quad \left(m^3/s\right)$$

Thus, the {complex} source strength/volume velocity of a physical monopole is:

$$\tilde{Q}_a = 4\pi a \frac{\tilde{B}}{z_o}\left[1-\frac{i}{ka}\right]e^{-ika} \quad \left(m^3/s\right)$$

Since $ka \ll 1$, then $1/ka \gg 1$, we can therefore approximate this expression as:

$$\tilde{Q}_a \simeq -4\pi i \cdot \frac{\tilde{B}}{z_o k}\Big\{\underbrace{\cos ka}_{\approx 1} - i\underbrace{\sin ka}_{\approx 0}\Big\} \simeq -4\pi i \cdot \frac{\tilde{B}}{z_o k} = -4\pi i \cdot \frac{\tilde{B}}{\rho_o ck} = -4\pi i \cdot \frac{\tilde{B}}{\rho_o \omega} \quad \left(m^3/s\right)$$

Thus, we see that: $\tilde{B} \simeq i\dfrac{\rho_o\omega}{4\pi}\tilde{Q}_a \quad \left(Pascal\text{-}m = kg/s^2\right)$

Expressed in terms of the complex source strength/volume velocity $\tilde{Q}_a \left(m^3/s\right)$ of the physical monopole sphere of radius *a*, the ***time-domain*** and ***frequency-domain*** complex pressure and radial particle velocity associated with this sound source are (for *r* > *a*):

$$\tilde{p}(r,t) = \frac{\tilde{B}}{r}e^{i(\omega t-kr)} = i\frac{\rho_o\omega}{4\pi}\frac{\tilde{Q}_a}{r}e^{i(\omega t-kr)} = \tilde{p}(r,\omega)e^{i\omega t} \quad \text{i.e.} \quad \tilde{p}(r,\omega) = i\frac{\rho_o\omega}{4\pi}\frac{\tilde{Q}_a}{r}e^{-ikr}$$

$$\tilde{u}_r(r,t) = \frac{1}{z_o}\frac{\tilde{B}}{r}\left[1-\frac{i}{kr}\right]e^{i(\omega t-kr)} = i\frac{\omega}{4\pi c}\frac{\tilde{Q}_a}{r}\left[1-\frac{i}{kr}\right]e^{i(\omega t-kr)} = \tilde{u}_r(r,\omega)e^{i\omega t} \quad \text{i.e.} \quad \tilde{u}_r(r,\omega) = i\frac{\omega}{4\pi c}\frac{\tilde{Q}_a}{r}\left[1-\frac{i}{kr}\right]e^{-ikr}$$