

Likewise, the real and imaginary parts of the complex radial acoustic energy flow velocity are:

$$c_{a_r}^r(r, \omega) = \text{Re}\{\tilde{c}_{a_r}(r, \omega)\} = c \frac{1}{\left[1 + (1/kr)^2\right]} \quad \text{and:} \quad c_{a_r}^i(r, \omega) = \text{Im}\{\tilde{c}_{a_r}(r, \omega)\} = c \frac{1/kr}{\left[1 + (1/kr)^2\right]}$$

The phase of the complex radial **specific** acoustic impedance and complex radial acoustic energy flow velocity is:

$$\varphi_{z_a} = \varphi_{c_a} = \Delta\varphi_{p-u} = \varphi_p - \varphi_u = \tan^{-1}(1/kr)$$

- (a) **In the “near” zone:** $kr \ll 1$, the complex radial **specific** acoustic impedance and the complex radial energy flow velocity are both largely **imaginary** (i.e. **reactive**):

$$\tilde{z}_{a_r}(r)/z_o = \tilde{c}_{a_r}(r)/c = [1 + i/kr] / \left[1 + (1/kr)^2\right] \sim i \cdot kr, \text{ increasing } \sim \text{linearly with } r, \text{ with}$$

fractional magnitude $|\tilde{z}_{a_r}(r)|/z_o = |\tilde{c}_{a_r}(r)|/c = 1/\sqrt{1 + (1/kr)^2} \sim kr \ll 1$ and phase

$\varphi_{z_a} = \varphi_{c_a} = \varphi_p - \varphi_u = \tan^{-1}(1/kr) \sim \tan^{-1}(\infty) \sim 90^\circ$. Again, the complex sound field $\tilde{S}(\vec{r}, t)$ in this region is **inertia**-like (i.e. **mass**-like), because $z_{a_r}^i(r) = \text{Im}\{\tilde{z}_{a_r}(r)\} > 0$ for $kr \ll 1$.

- (b) **In the “intermediate zone:** $kr \sim 1$, the complex radial **specific** acoustic impedance and the complex radial energy flow are both \sim an equal mix of **active** (i.e. real) and **reactive** (i.e. imaginary) components:

$$\tilde{z}_{a_r}(r)/z_o = \tilde{c}_{a_r}(r)/c = [1 + i/kr] / \left[1 + (1/kr)^2\right] \sim (1 + i)/2, \text{ with}$$

fractional magnitude $|\tilde{z}_{a_r}(r)|/z_o = |\tilde{c}_{a_r}(r)|/c = 1/\sqrt{1 + (1/kr)^2} \sim 1/\sqrt{2}$ and phase

$$\varphi_{z_a} = \varphi_{c_a} = \varphi_p - \varphi_u = \tan^{-1}(1/kr) \sim \tan^{-1}(1) \sim 45^\circ.$$

- (c) **In the “far” (or radiation) zone:** $kr \gg 1$, the complex radial **specific** acoustic impedance and the complex radial energy flow velocity are largely **real** (i.e. **active**):

$$\tilde{z}_{a_r}(r)/z_o = \tilde{c}_{a_r}(r)/c = 1/\left[1 + (1/kr)^2\right] \sim 1 \text{ with magnitude}$$

$|\tilde{z}_{a_r}(r)|/z_o = |\tilde{c}_{a_r}(r)|/c = 1/\sqrt{1 + (1/kr)^2} \sim 1$ and phase

$$\varphi_{z_a} = \varphi_{c_a} = \varphi_p - \varphi_u = \tan^{-1}(1/kr) \sim \tan^{-1}(0) \sim 0^\circ.$$

The **frequency domain** complex radial acoustic intensity associated with the point acoustic monopole also points outward in the radial (\hat{r}) direction; it and its magnitude are:

$$\tilde{I}_{a_r}(r, \omega) \equiv \frac{1}{2} \tilde{p}(r, \omega) \cdot \tilde{u}_r^*(r, \omega) = \frac{1}{2} \frac{1}{z_o} \frac{B_o^2}{r^2} \left[1 + \frac{i}{kr}\right] \quad \text{and:} \quad |\tilde{I}_{a_r}(r, \omega)| = \frac{1}{2} \frac{1}{z_o} \frac{B_o^2}{r^2} \sqrt{1 + (1/kr)^2}$$

The real and imaginary parts of the **frequency domain** radial complex acoustic intensity are:

$$I_{a_r}^r(r, \omega) = \text{Re}\{\tilde{I}_{a_r}(r, \omega)\} = \frac{1}{2} \frac{1}{z_o} \frac{B_o^2}{r^2} \quad \text{and:} \quad I_{a_r}^i(r, \omega) = \text{Im}\{\tilde{I}_{a_r}(r, \omega)\} = \frac{1}{2} \frac{1}{z_o} \frac{B_o^2}{r^2} \frac{1}{kr}$$