The relationships between the complex ***time-domain*** and complex ***frequency-domain*** over-pressure and radial particle velocity amplitudes for the point monopole sound source are:

$$\tilde{p}(r,t) = \frac{B_o}{r}\cdot e^{i(\omega t - kr)} = \tilde{p}(r,\omega)\cdot e^{i\omega t} \qquad i.e. \quad \tilde{p}(r,\omega) = \frac{B_o}{r}\cdot e^{-ikr}$$

$$\tilde{u}_{\mathrm{r}}(r,t) = \frac{1}{z_o}\frac{B_o}{r}\left[1 - \frac{i}{kr}\right]\cdot e^{i(\omega t - kr)} = \tilde{u}_{\mathrm{r}}(r,\omega)\cdot e^{i\omega t} \quad i.e. \quad \tilde{u}_{\mathrm{r}}(r,\omega) = \frac{1}{z_o}\frac{B_o}{r}\left[1 - \frac{i}{kr}\right]\cdot e^{-ikr}$$

Note that the purely real "amplitude" B_o is in general complex $\tilde{B} = |\tilde{B}|e^{i\varphi_B}\ (Pa\text{-}m)$, in order to accommodate an (arbitrary) absolute phase φ_B, which we can ***always*** "rotate" away, simply by re-defining the zero of time: $t \to t - (\varphi_B/\omega)$}. So, to make life simpler, we can set $\varphi_B = 0$, then $\tilde{B} = |\tilde{B}| = B_o$ is purely real. The ***magnitudes*** of the complex ***time-domain*** over-pressure and radial particle velocity amplitudes then are:

$$|\tilde{p}(r,t)| = \frac{|\tilde{B}|}{r} = |\tilde{p}(r,\omega)| \quad \text{and:} \quad |\tilde{u}_{\mathrm{r}}(r,t)| = \frac{1}{z_o}\frac{|\tilde{B}|}{r}\sqrt{1+(1/kr)^2} = |\tilde{u}_{\mathrm{r}}(r,\omega)|$$

The ***phases*** of the complex pressure and radial particle velocity are:

$$\varphi_p = \varphi_B = 0 \quad \text{and:} \quad \varphi_{u_{\mathrm{r}}} = \tan^{-1}(-1/kr) + \varphi_B = -\tan^{-1}(1/kr) + \varphi_B = -\tan^{-1}(1/kr)$$

Thus, we also see that: $\Delta\varphi_{p-u} \equiv \varphi_p - \varphi_{u_{\mathrm{r}}} = \varphi_B - \left(-\tan^{-1}(1/kr) + \varphi_B\right) = +\tan^{-1}(1/kr) \quad (= \varphi_z = \varphi_I)$

n.b. the phase difference $\Delta\varphi_{p-u} \equiv \varphi_p - \varphi_{u_{\mathrm{r}}}\ (= \varphi_z = \varphi_I)$ is independent of the absolute phase φ_B.

Let us examine the behavior of ***time-domain*** $\tilde{p}(r,t)$ and $\tilde{u}_{\mathrm{r}}(r,t)$ as *r* increases from *r* = 0. The complex over-pressure $\tilde{p}(r,t) = (B_o/r)e^{i(\omega t - kr)}$ simply decreases with increasing *r*, modulated by the {complex} oscillatory factor $e^{i(\omega t - kr)}$. The behavior of $\tilde{u}_{\mathrm{r}}(r,t) = \frac{1}{z_o}\frac{\tilde{B}}{r}\left[1 - \frac{i}{kr}\right]e^{i(\omega t - kr)}$ is mathematically more interesting… When $r \approx 0$ {*i.e.* in proximity to the point sound source}, then $kr \ll 1$, and the imaginary part of $\tilde{u}_{\mathrm{r}}(r,t)$ dominates, because then $|i/kr| = 1/kr \gg 1$, thus for $r \approx 0$ $(kr \ll 1)$, $\tilde{u}_{\mathrm{r}}(r,t)$ will be ~ 90° ***out-of-phase/in quadrature*** with the complex over-pressure $\tilde{p}(r,t)$, and hence in proximity to the point sound source, both the frequency domain complex radial specific acoustic impedance $\tilde{z}_a(r,\omega) = \tilde{p}(r,\omega)/\tilde{u}_{\mathrm{r}}(r,\omega)$ and the frequency domain complex radial acoustic intensity $\tilde{I}_a(r,\omega) = \frac{1}{2}\tilde{p}(r,\omega)\cdot\tilde{u}_{\mathrm{r}}^*(r,\omega)$ will be ***largely imaginary/reactive/non-propagating*** – *i.e.* for $r \approx 0$ $(kr \ll 1)$, sound energy is largely stored locally, oscillating/sloshing back-and-forth during each cycle of oscillation…