

For a given (angular) frequency ω , for each of the **propagating** duct eigenmodes $n \leq n_{cutoff}$, the **axial** component of the particle velocity $\tilde{u}_{n_x}(x, y, t) \hat{x} = \frac{1}{\omega \rho_o} \tilde{A}_n k_n \cos(n\pi y/a) e^{i(\omega t - k_n x)} \hat{x}$ is **in-phase** with the complex pressure $\tilde{p}_n(x, y, t) = \tilde{A}_n \cos(n\pi y/a) e^{i(\omega t - k_n x)}$, whereas the **transverse** component of the particle velocity $\tilde{u}_{n_y}(x, y, t) \hat{y} = \frac{-i}{\omega \rho_o} \tilde{A}_n (n\pi/a) \sin(n\pi y/a) e^{i(\omega t - k_n x)} \hat{y}$ is in **quadrature** (i.e. 90° out of phase) with the complex over-pressure amplitude.

The **total/net** complex **time-domain** 2-D particle velocity is likewise given by:

$$\vec{\tilde{u}}(x, y, t) = \sum_{n=0}^{\infty} \tilde{u}_n(x, y, t) = \frac{1}{\omega \rho_o} \sum_{n=0}^{\infty} \tilde{A}_n [k_n \cos(n\pi y/a) \hat{x} - i(n\pi/a) \sin(n\pi y/a) \hat{y}] e^{i(\omega t - k_n x)}$$

where: $k_x = k_n = \sqrt{k^2 - (n\pi/a)^2} = \sqrt{(\omega/c)^2 - (n\pi/a)^2}$ and: $n_{cutoff} = \text{int} \{ \omega a / \pi c \}$ (= floor $\{ \omega a / \pi c \}$).

The total/net complex pressure wave $\tilde{p}_{tot}(x, y, t) = \sum_{n=0}^{\infty} \tilde{p}_n(x, y, t)$ and 2-D particle velocity wave $\vec{\tilde{u}}(x, y, t) = \sum_{n=0}^{\infty} \tilde{u}_n(x, y, t)$ that propagate in a duct depend on the details of the coupling of the sound **source** to that duct. For example, a 2-D “line” monopole source of volumetric velocity **per unit length** $Q'_a = Q_a/L$ (m^2/s) located e.g. at $(x, y) = (0, y_o)$ in the duct will produce modal pressure **amplitudes** of:

$$\tilde{A}_n = \frac{\omega \rho_o Q'_a}{k_n a} \cos(n\pi y_o/a)$$

This relation predicts that the n^{th} modal pressure amplitude \tilde{A}_n becomes **infinite** at the cutoff frequency for that mode, $\omega_n^{cutoff} = n\pi c/a$ when $k_x = k_n = \sqrt{(\omega_{cutoff}/c)^2 - (n\pi/a)^2} = 0$!!! However, in the real world, nothing becomes infinite – e.g. the **finite** impedance of a real/physical acoustic source precludes infinite acoustic energy transfer to the duct. Nevertheless, the experimentally-measured modal pressure amplitudes \tilde{A}_n do indeed become large at/near the cutoff frequency!

The method of (an infinite set of) acoustic images can be used to model sound sources inside of (perfectly reflecting – even for partially reflecting) ducts – the planar walls of the duct act like mirrors, thus virtual “images” of the sound source in the duct are produced outside of the duct, as shown in the figure below: