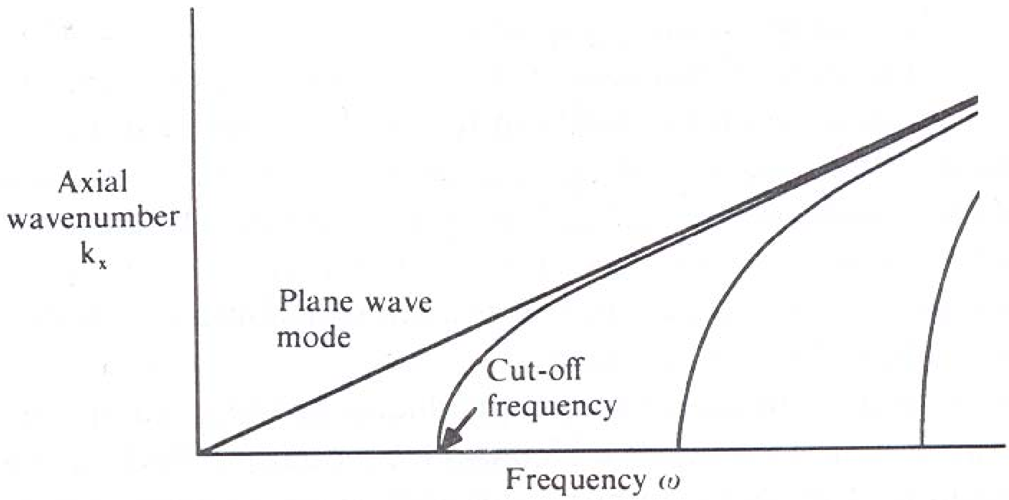


For a given (angular) frequency $\omega$, in general, the ***total/net complex*** pressure amplitude is a sum over ***all*** modes – the ***allowed/propagating*** individual complex pressure eigenmodes $n \le n_{cutoff}$, where $n_{cutoff}$ is the ***highest*** eigenmode number $n$ such that $k_x = k_n = \sqrt{(\omega/c)^2 - (n_{cutoff}\pi/a)^2} > 0$, *i.e.* $n_{cutoff} = \text{int}\{\omega a/\pi c\}$ $(= \text{floor}\{\omega a/\pi c\})$, **.and.** the individual ***non-propagating*** modes $n > n_{cutoff}$:

$$\tilde{p}_{tot}(x,y,t) = \sum_{n=0}^{\infty} \tilde{p}_n(x,y,t) = \sum_{n=0}^{\infty} \tilde{A}_n \cos(n\pi y/a) e^{i(\omega t - k_n x)}$$

{*n.b.* ***Far*** from the sound source, this reduces to the sum over ***propagating*** modes $n \le n_{cutoff}$ .}

The complex ***time-domain*** 2-D particle velocity $\vec{\tilde{u}}(\vec{r},t)$ associated with this problem is obtained via use of Euler's equation for inviscid fluid flow. Since $\tilde{p}(\vec{r},t) = \tilde{p}(x,y,t) \ne fcn(z)$, then $\vec{\nabla}\tilde{p}(\vec{r},t) = \vec{\nabla}\tilde{p}(x,y,t) = (\partial\tilde{p}(x,y,t)/\partial x)\hat{x} + (\partial\tilde{p}(x,y,t)/\partial y)\hat{y}$ , thus the particle velocity can ***only*** be in the $(x,y)$ direction(s), *i.e.* $\vec{u}(\vec{r},t) = \vec{u}(x,y,t) \ne fcn(z)$ since:

$$\frac{\partial \vec{u}(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla}p(\vec{r},t) \;\Rightarrow\; \frac{\partial \vec{\tilde{u}}(x,y,t)}{\partial t} = -\frac{1}{\rho_o}\left(\frac{\partial\tilde{p}(x,y,t)}{\partial x}\hat{x} + \frac{\partial\tilde{p}(x,y,t)}{\partial y}\hat{y}\right)$$

Euler's equation holds for each/every duct eigenmode *n*. With $\tilde{p}_n(x,y,t) = \tilde{A}_n \cos(n\pi y/a) e^{i(\omega t - k_n x)}$, the general form of the complex ***time-domain*** 2-D particle velocity for the $n^{th}$ duct eigenmode is thus:

$$\vec{\tilde{u}}_n(x,y,t) = \frac{1}{\omega\rho_o}\tilde{A}_n\left[k_n \cos(n\pi y/a)\hat{x} - i(n\pi/a)\sin(n\pi y/a)\hat{y}\right]e^{i(\omega t - k_n x)}$$

$$\text{where: } k_x = k_n = \sqrt{k^2 - (n\pi/a)^2} = \sqrt{(\omega/c)^2 - (n\pi/a)^2}$$