The boundary condition on the pressure at the two infinite, rigid parallel walls in the $\hat{y}$-direction is that there are pressure ***anti-nodes*** at $y = 0$ and $y = a$. Mathematically, this requires Neumann-type boundary conditions on the walls, *i.e.* $\partial\tilde{p}(x, y=0,t)/\partial y = \partial\tilde{p}(x, y=a,t)/\partial y = 0$, requiring cosine-type solutions for $\tilde{Y}(y) = e^{\mp ik_y y}$, i.e. $\tilde{Y}(y) \sim e^{ik_y y} + e^{-ik_y y} \sim \cos k_y y$ such that:

$$\cos k_y y\Big|_{y=0,a} = 1 \text{ with } \partial\cos k_y y/\partial y\Big|_{y=0,a} = \sin k_y y\Big|_{y=0,a} = 0 \;\Rightarrow\; k_y a = n\pi,\; n = 0,1,2,3...$$

Thus we see that: $k_y a = n\pi$ or: $k_y = n\pi/a$, $n = 0,1,2,3....$ Thus, for ***any*** given frequency $f = \omega/2\pi$, there are an ***infinite*** number of possible solutions (*aka* eigenmodes) for this wave equation, each one of the general form:

$$\tilde{p}_n(x,y,t) = \tilde{A}_n \cos(n\pi y/a)e^{i(\omega t - k_n x)} \text{ where: } k_x = \sqrt{k^2 - k_y^2} \;\Rightarrow\; k_n = \sqrt{k^2 - (n\pi/a)^2}$$

The transverse pressure distribution $\sim \cos(n\pi y/a)$ for $0 \le y \le a$ is a characteristic of the wall geometry associated with this problem – *i.e.* a ***duct***; and one which is caused by multiple, perfect (i.e lossless) reflections of the pressure waves off of the duct walls as they propagate in the $+\hat{x}$-direction. The integer *n* denotes the {duct-} mode of propagation.

The *n* = 0 mode is known as the ***axial*** plane-wave eigenmode of propagation. The $n \ge 1$ modes are collectively known as ***transverse*** duct eigenmodes. At a given frequency *f*, if a specific duct eigenmode *n* is excited, it may only propagate along the duct with a unique *axial* wavenumber given by $k_n = \sqrt{k^2 - (n\pi/a)^2}$.

Note that for each/every duct eigenmode *n* of propagation, there is an (angular) frequency ω for which the axial wavenumber $k_n = \sqrt{k^2 - (n\pi/a)^2} = \sqrt{(\omega/c)^2 - (n\pi/a)^2} = 0$. The so-called ***cutoff frequency*** for the n^{th} mode is: $\omega_n^{cutoff} = n\pi c/a$ or: $f_n^{cutoff} = nc/2a$. Below this cutoff frequency, the duct eigenmode *n* cannot propagate – it becomes an ***evanescent*** mode because the axial eigen-wavenumber k_n becomes purely ***imaginary*** for $f < f_n^{cutoff} = nc/2a$ – *i.e.* the duct eigenmode n is ***exponentially*** attenuated by a factor of $e^{-k_n x}$ when $f < f_n^{cutoff} = nc/2a$.

A plot of the ***dispersion curves*** - axial wavenumber $k_x = k_n$ *vs*. angular frequency ω showing the effect of the cutoff frequency *vs*. mode number *n* = 0, 1, 2, 3, … is shown in the figure below.