Since $p(\vec{r},t) = p(r,t) \neq fcn(\theta,\varphi)$, then $\vec{\nabla}p(\vec{r},t) = \vec{\nabla}p(r,t) = \left(\partial p(r,t)/\partial r\right)\hat{r}$ and thus the ***instantaneous/physical*** (*i.e.* purely ***real*** ***time-domain***) particle velocity $\vec{u}(\vec{r},t)$ associated with a spherically-symmetric monochromatic point sound source can ***only*** be oriented in the ***radial*** direction, *i.e.* $\vec{u}(\vec{r},t) = u_r(r,t)\hat{r} \neq fcn(\theta,\varphi)$:

$$\frac{\partial \vec{u}(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla}p(\vec{r},t) \;\Rightarrow\; \frac{\partial u_r(r,t)}{\partial t}\hat{r} = -\frac{1}{\rho_o}\frac{\partial p(r,t)}{\partial r}\hat{r} \;\Rightarrow\; \frac{\partial u_r(r,t)}{\partial t} = -\frac{1}{\rho_o}\frac{\partial p(r,t)}{\partial r}$$

Now: $\dfrac{\partial p(r,t)}{\partial r} = \dfrac{\partial}{\partial r}\left\{\dfrac{B_o}{r}\cos(\omega t - kr)\right\} = -\dfrac{B_o}{r^2}\cos(\omega t - kr) + \dfrac{kB_o}{r}\sin(\omega t - kr)$

Then: $\dfrac{\partial u_r(r,t)}{\partial t} = -\dfrac{1}{\rho_o}\dfrac{\partial p(r,t)}{\partial r} = -\dfrac{1}{\rho_o}\left\{-\dfrac{B_o}{r^2}\cos(\omega t - kr) + \dfrac{kB_o}{r}\sin(\omega t - kr)\right\}$

$$= +\frac{B_o}{\rho_o r}\left\{\frac{1}{r}\cos(\omega t - kr) - k\sin(\omega t - kr)\right\}$$

Thus:

$$u_r(r,t) = \frac{B_o}{\omega\rho_o r}\left\{\frac{1}{r}\sin(\omega t - kr) + k\cos(\omega t - kr)\right\} = \frac{B_o k}{\omega\rho_o r}\left\{\cos(\omega t - kr) + \frac{1}{kr}\sin(\omega t - kr)\right\} \;\; (m/s)$$

Using $c = \omega/k$ and $z_o \equiv \rho_o c$ this relation becomes:

$$u_r(r,t) = \frac{B_o}{z_o r}\left\{\cos(\omega t - kr) + \frac{1}{kr}\sin(\omega t - kr)\right\} \;\; (m/s)$$

We can then "complexify" the radial-outgoing spherical over-pressure and particle velocity waves:

$$p(r,t) = \frac{B_o}{r}\cos(\omega t - kr) \;\Rightarrow\; \tilde{p}(r,t) = \frac{B_o}{r}\left\{\cos(\omega t - kr) + i\sin(\omega t - kr)\right\} = \frac{B_o}{r}e^{i(\omega t - kr)}$$

and: $u_r(r,t) = \dfrac{1}{z_o}\dfrac{B_o}{r}\left\{\cos(\omega t - kr) + \dfrac{1}{kr}\sin(\omega t - kr)\right\} \;\Rightarrow$

$$\tilde{u}_r(r,t) = \frac{1}{z_o}\frac{B_o}{r}\left[\left\{\cos(\omega t - kr) + \frac{1}{kr}\sin(\omega t - kr)\right\} + i\left\{\sin(\omega t - kr) - \frac{1}{kr}\cos(\omega t - kr)\right\}\right]$$

$$= \frac{1}{z_o}\frac{B_o}{r}\left[\left\{\cos(\omega t - kr) + i\sin(\omega t - kr)\right\} - i\frac{1}{kr}\left\{\cos(\omega t - kr) + i\sin(\omega t - kr)\right\}\right]$$

$$= \frac{1}{z_o}\frac{B_o}{r}\left[\left\{e^{i(\omega t - kr)}\right\} - i\frac{1}{kr}\left\{e^{i(\omega t - kr)}\right\}\right] = \frac{1}{z_o}\frac{B_o}{r}\left[1 - \frac{i}{kr}\right]e^{i(\omega t - kr)} \quad \left\{ = \frac{1}{z_o}\left[1 - \frac{i}{kr}\right]\tilde{p}(r,t)\right\}$$