Using the relations $\tilde{z}_{a_z} = \rho_o \tilde{c}_{a_z}$ and $z_o = \rho_o c$ the above relation can be rewritten as a dimensionless quantity:

$$\frac{\tilde{z}_{a_z}(r=z,\omega)}{z_o} = \frac{\left[1-e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]}{\left[1-\frac{1}{\sqrt{1+(a/z)^2}}e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]} = \frac{\tilde{c}_{a_z}(r=z,\omega)}{c}$$

The ***frequency-domain on-axis*** complex longitudinal acoustic intensity is:

$$\begin{aligned}\tilde{I}_{a_z}(r=z,\omega) &= \tfrac{1}{2}\tilde{p}(r=z,\omega)\tilde{u}_z^*(r=z,\omega)\\ &= \tfrac{1}{2}p_o u_o\left[1-e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\left[1-\frac{1}{\sqrt{1+(a/z)^2}}e^{+ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\\ &= \tfrac{1}{2}I_o\left[1-e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\left[1-\frac{1}{\sqrt{1+(a/z)^2}}e^{+ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\\ &= I_o^{rms}\left[1-e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\left[1-\frac{1}{\sqrt{1+(a/z)^2}}e^{+ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\end{aligned}$$

We leave it as an exercise for the interested/motivated reader to explicitly obtain the real and imaginary parts of ***on-axis*** $\tilde{z}_a^{\parallel}(r=z,\omega)$ and $\tilde{I}_a^{\parallel}(r=z,\omega)$, calculate the {purely real} ***on-axis*** potential/kinetic/total acoustic energy densities, *etc*.

Please see/look at plots of the above complex acoustic quantities associated with the baffled plane circular piston, available on the Physics 406 Software web-page, at the following URL:

http://courses.physics.illinois.edu/phys406/406pom_sw.html