The real and imaginary components of the ***on-axis*** ***frequency-domain*** complex over-pressure are:

$$p_{\mathrm{r}}(r=z,\omega)=\mathrm{Re}\{\tilde{p}(r=z,\omega)\}=\ \ p_o\left[\cos kz-\cos kz\sqrt{1+(a/z)^2}\right]$$

$$p_{\mathrm{i}}(r=z,\omega)=\mathrm{Im}\{\tilde{p}(r=z,\omega)\}=-p_o\left[\sin kz-\sin kz\sqrt{1+(a/z)^2}\right]$$

The ***magnitude*** of the ***on-axis*** ***frequency-domain*** complex over-pressure amplitude is:

$$|\tilde{p}(r=z)|=\sqrt{\tilde{p}(r=z)\,\tilde{p}^*(r=z)}=\sqrt{2}\,p_o\left\{1-\cos kz\left[\sqrt{1+(a/z)^2}-1\right]\right\}^{1/2}$$

Using the trigonometric identity $\cos 2A=1-2\sin^2 A$, this can be equivalently written as:

$$|\tilde{p}(r=z)|=p_o\left|\sin\left\{\tfrac{1}{2}kz\left[\sqrt{1+(a/z)^2}-1\right]\right\}\right|$$

For $z\gg a$, the Taylor series expansion for $\sqrt{1+(a/z)^2}\simeq 1+\frac{1}{2}(a/z)^2$ **.and.** if ***additionally*** $z\gg\frac{1}{2}ka^2=\frac{1}{2}(2\pi/\lambda)a^2=\pi a^2/\lambda\equiv Rayleigh\ length$, then the ***magnitude*** of the "***far-field***" ***on-axis*** ***frequency-domain*** complex over-pressure amplitude is:

$$|\tilde{p}_{far}(r=z)|\simeq\tfrac{1}{2}p_o(a/z)ka=p_o\left(\pi a^2/\lambda z\right)$$

Note that the ***magnitude*** of the "***far-field***" ***on-axis*** ***frequency-domain*** complex over-pressure is bounded by:

$$0\le|\tilde{p}_{far}(r=z)|\le 2p_o \ \text{ for } 0\le r=z\le\infty$$

Extrema of $|\tilde{p}(r=z)|$ occur when: $\frac{1}{2}kz\left[\sqrt{1+(a/z)^2}-1\right]=\frac{1}{2}m\pi,\quad m=0,1,2,3,4,5,\ldots$ Solving the quadratic equation for *z*, $|\tilde{p}(r=z)|$ extrema occur when: $z_m/a=(a/m\lambda)-(m\lambda/4a)$, $m=1,2,3,4,5,\ldots$

Coming in from $z=\infty$, the first ***maxima*** $|\tilde{p}(r=z_1)|_{\max}$ occurs at: $z_1/a=(a/\lambda)-(\lambda/4a)$, the next ***maxima*** $|\tilde{p}(r=z_3)|_{\max}$ occurs at: $z_3/a=(a/3\lambda)-(3\lambda/4a)$, and so on. Zeroes of $|\tilde{p}(r=z_m)|$ occur in between the local ***maxima***, when $m=2,4,6,8,\ldots$

Thus, having obtained the ***on-axis*** complex ***time-domain*** over-pressure amplitude $\tilde{p}(r=z,t)$, we next use the {linearized} Euler's equation for inviscid fluid flow to obtain the ***on-axis*** complex ***time-domain*** particle velocity $\tilde{\vec{u}}(r=z,\omega,t)$:

$$\frac{\partial\tilde{\vec{u}}(r=z,t)}{\partial t}=-\frac{1}{\rho_o}\vec{\nabla}\tilde{p}(r=z,t)=-\frac{1}{\rho_o}\frac{\partial\tilde{p}(r=z,t)}{\partial z}\hat{z}$$