$$\tilde{p}_{bafl}(r,t) = 2\tilde{p}_{un\text{-}bafl}(r,t) = 2\frac{\tilde{B}}{r}e^{i(\omega t-kr)} = 2i\frac{\rho_o\omega}{4\pi}\frac{\tilde{Q}_a}{r}e^{i(\omega t-kr)} = i\frac{\rho_o\omega}{2\pi}\frac{\tilde{Q}_a}{r}e^{i(\omega t-kr)} = ip_o(r,\omega)e^{i(\omega t-kr)}$$

The ***amplitude*** of the complex over-pressure a distance r away from the ***baffled*** point sound source {located at the origin} is: $\tilde{p}_o(r,\omega) \equiv \rho_o\omega\tilde{Q}_a/2\pi r$.

Hence, the ***amplitude*** of the complex over-pressure a distance r' away from the ***baffled infinitesimal*** point sound source associated with the infinitesimal area element dS located at the point $\vec{\rho}$ on the surface of the plane circular piston {see figure above} is: $\tilde{p}_o(r',\omega) = \rho_o\omega\tilde{Q}_a/2\pi r'$.

Hence the ***infinitesimal contribution*** to the complex ***time-domain*** over-pressure amplitude a distance r' away from the ***baffled infinitesimal*** point sound source associated with the infinitesimal area element dS located at the point $\vec{\rho}$ on the surface of the plane circular piston is: $dp(r',\omega) = \rho_o\omega dQ/2\pi r' = \rho_o\omega u_o dS/2\pi r'$ where in the last step, we used the infinitesimal relation $dQ = u_o dS$.

The corresponding ***infinitesimal contribution*** to the complex ***time-domain*** over-pressure is:

$$d\tilde{p}(r',t) = dp(r',t)e^{i(\omega t-kr')} = i\frac{\rho_o\omega}{2\pi}\frac{dQ}{r'}e^{i(\omega t-kr')} = i\frac{\rho_o\omega}{2\pi}\frac{u_o}{r'}e^{i(\omega t-kr')}dS$$

Then, we simply need to ***sum up*** all of the individual infinitesimal ***contribution(s)*** $d\tilde{p}(r',t)$ – *i.e.* we need to ***integrate*** $d\tilde{p}(r',t)$ over the area of the plane circular piston, in order to obtain the ***total*** complex ***time-domain*** over-pressure ***amplitude*** a distance r away from the ***center*** of the ***baffled*** harmonically oscillating plane circular piston, {which is taken to be the local origin of coordinates – see above figure}:

$$\tilde{p}(r,t) = \int d\tilde{p}(r't) = \int i\frac{\rho_o\omega}{2\pi}\frac{u_o}{r'}e^{i(\omega t-kr')}dS = i\frac{\rho_o\omega u_o}{2\pi}\int\frac{1}{r'}e^{i(\omega t-kr')}dS$$

Referring to the above figure, we need to express r' in terms of r, ρ and angles θ and φ. Vectorially $\vec{r} = \vec{\rho} + \vec{r}'$, hence $\vec{r}' = \vec{r} - \vec{\rho}$, then:

$$r'^2 = |\vec{r}'|^2 = \vec{r}'\bullet\vec{r}' = (\vec{r}-\vec{\rho})\bullet(\vec{r}-\vec{\rho}) = \vec{r}\bullet\vec{r} - 2\vec{\rho}\bullet\vec{r} + \vec{\rho}\bullet\vec{\rho} = |\vec{r}|^2 - 2\vec{\rho}\bullet\vec{r} + |\vec{\rho}|^2 = r^2 - 2\vec{\rho}\bullet\vec{r} + \rho^2$$

The vector $\vec{\rho}$ lies in the *x-y* plane, and has components: $\vec{\rho} = \rho\hat{\rho} = \rho_x\hat{x} + \rho_y\hat{y} = \rho\cos\varphi\hat{x} + \rho\sin\varphi\hat{y}$. It has ***magnitude***: $\rho = |\vec{\rho}| = \sqrt{\rho_x^2 + \rho_y^2} = \sqrt{\rho^2\cos^2\varphi + \rho^2\sin^2\varphi} = \rho\sqrt{\cos^2\varphi + \sin^2\varphi} = \rho$.

The vector $\vec{r}$ {*n.b.* ***fixed*** in space} has components {for the observation/listener position at $\vec{r}$ lying ***off-axis***, somewhere in the *x-z* plane}: $\vec{r} = r_x\hat{x} + 0 + r_z\hat{z} = x\hat{x} + z\hat{z} = r\sin\theta\hat{x} + r\cos\theta\hat{z}$ and has ***magnitude***: $r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{x^2 + y^2} = \sqrt{r^2\sin^2\theta + r^2\cos^2\theta} = r\sqrt{\sin^2\theta + \cos^2\theta} = r$