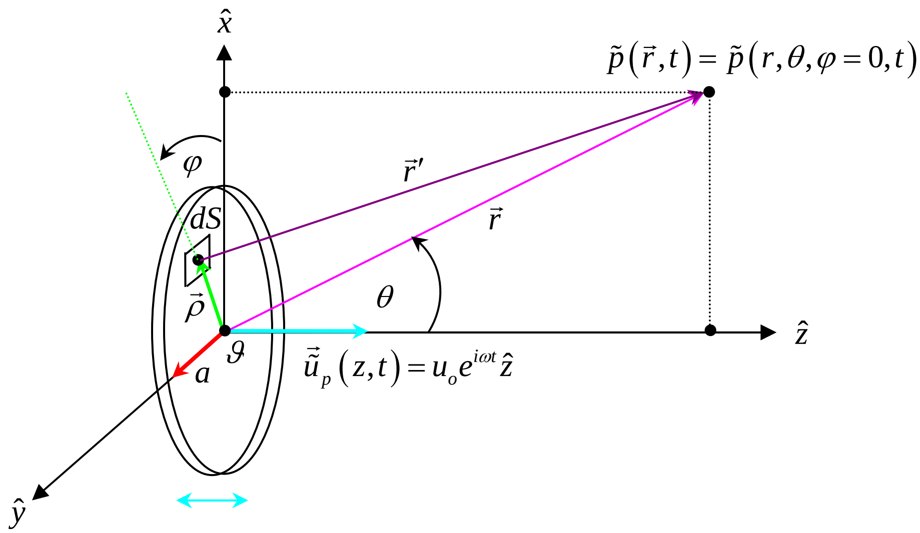
Example # 6: The Plane Circular Piston (A Simple Acoustics Model of a Loudspeaker):

Consider a longitudinally oscillating piston of radius *a* mounted on an infinite, perfectly rigid {*i.e.* immovable} planar baffle {oriented in the *x*-*y* plane at *z* = 0} as shown in the figure below.



The surface of the plane circular piston oscillates harmonically back and forth along the $\hat{z}$ -axis with complex velocity $\vec{\tilde{u}}_p(z,t)=u_o e^{i\omega t}\hat{z}$. What is the resulting complex pressure $\tilde{p}(\vec{r},t)=\tilde{p}(r,\theta,\varphi=0,t)$ *e.g.* at an observation/listener point $\vec{r}=(r,\theta,\varphi=0)$ lying in the *x*-*z* plane?

Note that the harmonically oscillating baffled plane circular piston is a ***spatially-extended*** sound source, *i.e.* it is ***not*** a ***point*** sound source. We can consider the harmonically oscillating baffled plane circular piston as a ***coherent*** linear superposition of infinitesimal, ***point*** sound sources, each of which ***isotropically*** radiate sound waves into the ***forward*** hemisphere $\{\hat{z}>0\}$ {since the piston is mounted on an infinite, rigid baffle lying in the *x*-*y* plane at *z* = 0}. Thus, each ***infinitesimal*** area element dS on the plane circular piston acts like a ***point*** sound source, with infinitesimal ***volume velocity strength*** $dQ=u_o dS\ \left(m^3/s\right)$.

From **Example # 4** above, we learned that the complex ***time-domain*** over-pressure amplitude associated with an ***unbaffled physical*** point sound source {located at the origin} of volume velocity strength $\tilde{Q}_a$ harmonically radiating sound waves isotropically into 4π steradians (with ***directivity*** $\mathbb{Q}=1$) a distance r away from this sound source is:

$$\tilde{p}_{un\text{-}bafl}(r,t)=\frac{\tilde{B}}{r}e^{i(\omega t-kr)}=i\frac{\rho_o\omega}{4\pi}\frac{\tilde{Q}_a}{r}e^{i(\omega t-kr)}$$

However, for a ***baffled*** physical point sound source {located at the origin} of volume velocity strength $\tilde{Q}_a$ harmonically radiating sound waves isotropically ***only*** into the 2π steradians of the $\{\hat{z}>0\}$ forward hemisphere (with ***directivity*** $\mathbb{Q}=2$) the complex ***time-domain*** over-pressure a distance *r* away from this sound source is $2\times$ this, *i.e.*: