$$\vec{\tilde{u}}_{tot}(\vec{r},t) = \frac{\tilde{B}}{z_o}\left\{\frac{1}{r_1}\left[1-\frac{i}{kr_1}\right]e^{-ikr_1}\hat{r}_1 - \frac{1}{r_2}\left[1-\frac{i}{kr_2}\right]e^{-ikr_2}\hat{r}_2\right\}\cdot e^{i\omega t} \quad \text{using: } \tilde{B} \simeq i\frac{\rho_o\omega}{4\pi}\tilde{Q}_a \text{ and: } z_o = \rho_o c$$

$$= i\frac{\omega\tilde{Q}_a}{4\pi c}\left\{\frac{1}{r_1}\left[1-\frac{i}{kr_1}\right]e^{-ikr_1}\hat{r}_1 - \frac{1}{r_2}\left[1-\frac{i}{kr_2}\right]e^{-ikr_2}\hat{r}_2\right\}\cdot e^{i\omega t} = \vec{\tilde{u}}_{tot}(\vec{r},\omega)e^{i\omega t}$$

The acoustic monopole sources, of source strength/volume velocity $\pm\tilde{Q}_a$ are located at $\vec{d}_1 = +d\hat{z}$ and $\vec{d}_2 = -d\hat{z}$ with $|\vec{d}_1| = |\vec{d}_2| = d$. Vectorially, we see that $\vec{r} = \vec{d}_1 + \vec{r}_1$ and also that $\vec{r} = \vec{d}_2 + \vec{r}_2$, with $\vec{r} = r\hat{r}$ and $|\vec{r}| = r = \sqrt{x^2+y^2+z^2}$. In Cartesian coordinates $\hat{r} = \sin\theta\cos\varphi\hat{x} + \sin\theta\sin\varphi\hat{y} + \cos\theta\hat{z}$. We also see that: $\vec{r}_1 = \vec{r} - \vec{d}_1$ and $\vec{r}_2 = \vec{r} - \vec{d}_2$. Using the law of cosines $c^2 = a^2 + b^2 - 2ab\cos\theta$:
$|\vec{r}_1| = r_1 = \sqrt{r^2 + d^2 - 2rd\cos\theta}$ and $|\vec{r}_2| = r_2 = \sqrt{r^2 + d^2 - 2rd\cos(\pi-\theta)} = \sqrt{r^2 + d^2 + 2rd\cos\theta}$, with $\vec{r}_1 = |\vec{r}_1|\hat{r}_1 = r_1\hat{r}_1 = \vec{r} - \vec{d}_1 = r\hat{r} - d\hat{z}$ and $\vec{r}_2 = |\vec{r}_2|\hat{r}_2 = r_2\hat{r}_2 = \vec{r} - \vec{d}_2 = r\hat{r} + d\hat{z}$ with $\hat{r}_1 = (r\hat{r} - d\hat{z})/r_1$ and $\hat{r}_2 = (r\hat{r} + d\hat{z})/r_2$.

Since $\hat{r}_1 \neq \hat{r}_2 \neq \hat{r}$ (especially in the "near-field" region), the above expression for the total / resultant complex ***vector*** particle velocity $\vec{\tilde{u}}_{tot}(\vec{r},t)$ is not easy to evaluate, analytically. However, note that it does simplify considerably in the "far"-field region, where $\hat{r}_1 \simeq \hat{r}_2 \simeq \hat{r}$ and $r_1 \simeq r_2 \simeq r \gg d$.

It is quite clear from the above formulae for $\tilde{p}_{tot}(\vec{r},t)$ and $\vec{\tilde{u}}_{tot}(\vec{r},t)$ {as well as from previous P406 lectures on sound interference effects with 2 (or more) sources} that interference effects will indeed manifest themselves here in this situation, albeit in a much more complicated manner….

However, the nature of this problem is such that ***all*** of the above quantities that we have calculated analytically e.g. for the various simpler sound sources can be also easily coded up on a computer, e.g. using MATLAB, Mathematica or *e.g.* a C/C++ based-program coupled to a graphics software package for plots, not just for them, but for this problem as well…

Computational calculations can also be done for {arbitrarily} higher-order acoustic multipoles – *e.g.* linear {and/or crossed (*aka* lateral)} quadrupoles (the tuning fork is an example of a linear quadrupole), sextupoles, octupoles, hexadecapoles, arbitrary linear 1-D acoustic arrays, 2-D/3-D acoustic arrays, all using the principle of linear superposition for N monopole sound sources… an arbitrary sound source can always be decomposed into a linear combination of acoustic multipoles, with suitably chosen complex coefficients – the multipole strengths…

Some very nice animation demos of pressure fields for monopole, dipole, quadrupole… sound sources exist *e.g.* at Prof. Dan Russell's website:

http://www.acs.psu.edu/drussell/Demos/rad2/mdq.html