

The complex radial **specific** acoustic impedance and energy flow velocity are (unchanged):

$$\frac{\tilde{z}_a(r, \omega)}{z_o} = \frac{1}{[1 - i/kr]} = \frac{[1 + i/kr]}{[1 + (1/kr)^2]} = \frac{\tilde{c}_a(r, \omega)}{c}$$

The **frequency-domain** complex radial acoustic intensity (for  $r > a$ ) is:

$$\tilde{I}_a(r, \omega) \equiv \frac{1}{2} \tilde{p}(r, \omega) \tilde{u}_r^*(r, \omega) = \frac{1}{2} \frac{1}{z_o} \frac{|\tilde{B}|^2}{r^2} \left[ 1 + \frac{i}{kr} \right] = \frac{\rho_o \omega^2}{32\pi^2 c} \frac{|\tilde{Q}_a|^2}{r^2} \left[ 1 + \frac{i}{kr} \right]$$

Note that  $\tilde{I}_a(r, \omega) \propto f^2$  - *i.e.* an acoustic monopole sound source is **not** efficient at very low frequencies in terms of generating sound....

The **frequency-domain** complex acoustic power associated with the physical monopole sound source (for  $r > a$ ) is:

$$\tilde{P}_a(r, \omega) = \int_S \tilde{I}_a(r, \omega) \hat{r} \cdot d\vec{S} = 2\pi \frac{|\tilde{B}|^2}{z_o} \left[ 1 + \frac{i}{kr} \right] = \rho_o \omega^2 \frac{|\tilde{Q}_a|^2}{8\pi c} \left[ 1 + \frac{i}{kr} \right]$$

The **frequency-domain** potential, kinetic and total energy densities associated with the physical monopole sound source (for  $r > a$ ) are:

$$w_a^{potl}(r, \omega) \equiv \frac{1}{4} \frac{|\tilde{p}(r, \omega)|^2}{\rho_o c^2} = \frac{\rho_o \omega^2}{64\pi^2 c^2} \frac{|\tilde{Q}_a|^2}{r^2} \quad (\text{Joules}/m^3)$$

$$w_a^{kin}(r, \omega) \equiv \frac{1}{4} \rho_o (\tilde{u}_r(r, \omega) \cdot \tilde{u}_r^*(r, \omega)) = \frac{1}{4} \rho_o |\tilde{u}_r(r, \omega)|^2 = \frac{\rho_o \omega^2}{64\pi^2 c^2} \frac{|\tilde{Q}_a|^2}{r^2} \left[ 1 + \left( \frac{1}{kr} \right)^2 \right] \quad (\text{Joules}/m^3)$$

$$w_a^{tot}(r, \omega) \equiv w_a^{potl}(r, \omega) + w_a^{kin}(r, \omega) = \frac{\rho_o \omega^2}{64\pi^2 c^2} \frac{|\tilde{Q}_a|^2}{r^2} \left[ 2 + \left( \frac{1}{kr} \right)^2 \right] \quad (\text{Joules}/m^3)$$