Note also that the 3-D {1-D} delta functions $\delta^3(\vec{r})$ { $\delta(r)$ } have SI units of m^{-3} { m }, respectively; the integrals $\int_V \delta^3(\vec{r})\,dV = 1$ and $\int_{r=0}^{r=\infty} \delta(r)\,dr = 1$ respectively, are ***dimensionless***.

If we now ***integrate*** the above ***inhomogeneous*** 2nd order linear differential equation over a {finite} arbitrary volume V but *e.g.* centered on, and thus containing the origin $\vec{r} = 0$, where the isotropic point sound source is located:

$$\int_V \nabla^2 p(\vec{r},t)\,dV - \frac{1}{c^2}\int_V \frac{\partial^2 p(\vec{r},t)}{\partial t^2}\,dV = -4\pi B_o \int_V \delta^3(\vec{r})\,dV \cos\omega t$$

Then using the ***Gauss divergence theorem***: $\int_V \nabla^2 p(\vec{r},t)\,dV = \int_V \vec{\nabla}\bullet\vec{\nabla} p(\vec{r},t)\,dV = \int_S \vec{\nabla} p(\vec{r},t)\bullet\hat{n}\,dS$ where $\hat{n}$ is the outward-pointing unit normal to the surface S {which encloses/bounds the volume V} and: $\int_V \delta^3(\vec{r})\,dV = 1$, the above integral relation then becomes:

$$\int_S \vec{\nabla} p(\vec{r},t)\bullet\hat{n}\,dS - \frac{1}{c^2}\int_V \frac{\partial^2 p(\vec{r},t)}{\partial t^2}\,dV = -4\pi B_o \cos\omega t$$

A ***spherically-symmetric*** time-dependent scalar $p(\vec{r},t)$ over-pressure has rotational invariance/rotational symmetry and therefore ***cannot*** have any explicit θ- and/or φ-dependence – only *r*-dependence. Thus $p(\vec{r},t) = p(r,t) \neq fcn(\theta,\varphi)$, and hence for a spherically-symmetric point sound source located at the origin of coordinates, then:

$$\vec{\nabla} p(\vec{r},t) = \vec{\nabla} p(r,t) = \left\{\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\hat{\varphi}\right\} p(r,t) = \frac{\partial p(r,t)}{\partial r}\hat{r}$$

and the 3-D integral wave equation for the scalar over-pressure $p(\vec{r},t)$ associated with this isotropic point sound source (for $r > 0$) becomes:

$$\int_S \frac{\partial p(\vec{r},t)}{\partial r}\hat{r}\bullet\hat{n}\,dS - \frac{1}{c^2}\int_V \frac{\partial^2 p(\vec{r},t)}{\partial t^2}\,dV = -4\pi B_o \cos\omega t$$

The ***instantaneous/physical*** (*i.e.* purely ***real*** ***<u>time</u>***-***<u>domain</u>***) solution to this integral wave equation is a purely ***<u>real</u>*** spherical-outgoing harmonic over-pressure wave:

$p(r,t) = \frac{B_o}{r}\cos(\omega t - kr)$ $(Pascals)$. Note that the constant B_o has *SI* units of $Pascal\text{-}m$.

The ***instantaneous/physical*** (*i.e.* purely ***real*** ***<u>time</u>***-***<u>domain</u>***) particle velocity $\vec{u}(\vec{r},t)$ associated with this problem is determined via use of the {linearized} Euler's equation for inviscid fluid flow:

$$\frac{\partial \vec{u}(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla} p(\vec{r},t)$$