$$\varphi_{p_{tot}}(x) \equiv \tan^{-1}\left(\frac{\mathrm{Im}\{\tilde{p}_{tot}(x,t)\}}{\mathrm{Re}\{\tilde{p}_{tot}(x,t)\}}\right) = \tan^{-1}\left(\frac{\mathrm{Im}\left\{\left[1 + |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]\right\}}{\mathrm{Re}\left\{\left[1 + |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]\right\}}\right)$$

$$= \tan^{-1}\left[\frac{\sin kx\left(1+|\tilde{R}|\cos\left(2kx+\Delta\varphi_{BA}^{o}\right)\right)+|\tilde{R}|\cos kx\cdot\sin\left(2kx+\Delta\varphi_{BA}^{o}\right)}{\cos kx\left(1+|\tilde{R}|\cos\left(2kx+\Delta\varphi_{BA}^{o}\right)\right)-|\tilde{R}|\sin kx\cdot\sin\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]$$

and:

$$\varphi_{u_{tot}}(x) \equiv \tan^{-1}\left(\frac{\mathrm{Im}\{\tilde{u}_{tot}^{\|}(x,t)\}}{\mathrm{Re}\{\tilde{u}_{tot}^{\|}(x,t)\}}\right) = \tan^{-1}\left(\frac{\mathrm{Im}\left\{\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]\right\}}{\mathrm{Re}\left\{\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]\right\}}\right)$$

$$= \tan^{-1}\left[\frac{\sin kx\left(1-|\tilde{R}|\cos\left(2kx+\Delta\varphi_{BA}^{o}\right)\right)-|\tilde{R}|\cos kx\cdot\sin\left(2kx+\Delta\varphi_{BA}^{o}\right)}{\cos kx\left(1-|\tilde{R}|\cos\left(2kx+\Delta\varphi_{BA}^{o}\right)\right)+|\tilde{R}|\sin kx\cdot\sin\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]$$

The complex longitudinal ***specific*** acoustic impedance associated with the two counter-propagating 1-D monochromatic plane waves is:

$$\tilde{z}_{a\,tot}^{\|}(x) \equiv \frac{\tilde{p}_{tot}(x,t)}{\tilde{u}_{tot}^{\|}(x,t)} = \frac{\cancel{\tilde{A}}\left[1 + |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]\cdot \cancel{e^{i(\omega t-kx)}}}{\frac{\cancel{\tilde{A}}}{\rho_o c}\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]\cdot \cancel{e^{i(\omega t-kx)}}} = \rho_o c\frac{\left[1 + |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]}{\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]}$$

Since the ***characteristic*** ***longitudinal*** ***specific*** acoustic impedance of "***free air***" is $z_o \equiv \rho_o c$, then:

$$\tilde{z}_{a\,tot}^{\|}(x) = z_o\frac{\left[1 + |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]}{\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]} = z_o\frac{\left[1 + |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]}{\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]}\cdot\frac{\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]^{*}}{\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]^{*}}$$

$$= z_o\frac{\left[1 + |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]}{\left[1 - |\tilde{R}| e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]}\cdot\frac{\left[1 - |\tilde{R}| e^{-i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]}{\left[1 - |\tilde{R}| e^{-i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right]} = z_o\frac{\left[1 + |\tilde{R}| e^{i(2kx+\Delta\varphi_{BA})} - |\tilde{R}| e^{-i\left(2kx+\Delta\varphi_{BA}^{o}\right)} - |\tilde{R}|^{2}\right]}{\left[1 - |\tilde{R}| e^{i(2kx+\Delta\varphi_{BA})} - |\tilde{R}| e^{-i\left(2kx+\Delta\varphi_{BA}^{o}\right)} + |\tilde{R}|^{2}\right]}$$

$$= z_o\frac{\left[1 + |\tilde{R}|\left\{e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)} - e^{-i\left(2kx+\Delta\varphi\varphi_{BA}^{o}\right)}\right\} - |\tilde{R}|^{2}\right]}{\left[1 - |\tilde{R}|\left\{e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)} + e^{-i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right\} + |\tilde{R}|^{2}\right]} = z_o\frac{\left[\left\{1 - |\tilde{R}|^{2}\right\} + |\tilde{R}|\left\{e^{i\left(2kx+\Delta\varphi_{BA}^{o}\right)} - e^{-i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right\}\right]}{\left[\left\{1 + |\tilde{R}|^{2}\right\} - |\tilde{R}|\left\{e^{i\left(2kx+\varphi_{BA}^{o}\right)} + e^{-i\left(2kx+\Delta\varphi_{BA}^{o}\right)}\right\}\right]}$$