and:

$$\begin{split} \left| \tilde{u}_{tot}^{\parallel} \left(x, t \right) \right| &= \sqrt{\tilde{u}_{tot}^{\parallel}} \left(x, t \right) \cdot \tilde{u}_{tot}^{\parallel *} \left(x, t \right) \\ &= \frac{\left| \tilde{A} \right|}{\rho_o c} \sqrt{\left(1 - \left| \tilde{R} \right| e^{i\left(2kx + \Delta \varphi_{BA}^o \right)} \right) \cdot \left(1 - \left| \tilde{R} \right| e^{i\left(2kx + \Delta \varphi_{BA}^o \right)} \right)^*} \\ &= \frac{\left| \tilde{A} \right|}{\rho_o c} \sqrt{\left(1 - \left| \tilde{R} \right| e^{i\left(2kx + \Delta \varphi_{BA}^o \right)} \right) \cdot \left(1 - \left| \tilde{R} \right| e^{-i\left(2kx + \Delta \varphi_{BA}^o \right)} \right)} \\ &= \frac{\left| \tilde{A} \right|}{\rho_o c} \sqrt{1 - \left| \tilde{R} \right| e^{i\left(2kx + \Delta \varphi_{BA}^o \right)} - \left| \tilde{R} \right| e^{-i\left(2kx + \Delta \varphi_{BA}^o \right)} + \left| \tilde{R} \right|^2} \\ &= \frac{\left| \tilde{A} \right|}{\rho_o c} \sqrt{1 - \left| \tilde{R} \right| \left\{ e^{i\left(2kx + \Delta \varphi_{BA}^o \right)} + e^{-i\left(2kx + \Delta \varphi_{BA}^o \right)} \right\} + \left| \tilde{R} \right|^2} \\ &= \frac{\left| \tilde{A} \right|}{\rho_o c} \sqrt{1 - 2\left| \tilde{R} \right| \cos\left(2kx + \Delta \varphi_{BA}^o \right) + \left| \tilde{R} \right|^2} \end{split}$$

Thus, *e.g.* for an observer/listener's position x = 0, **.and.** for <u>*equal-strength*</u> over-pressure amplitudes $|\tilde{A}| = |\tilde{B}| \implies |\tilde{R}| \equiv |\tilde{B}| / |\tilde{A}| = 1$ (*i.e.* a *pure* standing wave!) these formulae simplify to:

$$\left|\tilde{p}_{tot}\left(x=0,t\right)\right| = \sqrt{2}\left|\tilde{A}\right| \sqrt{1+\cos\Delta\varphi_{BA}^{o}} \text{ and: } \left|\tilde{u}_{tot}^{\parallel}\left(x=0,t\right)\right| = \frac{\sqrt{2}\left|\tilde{A}\right|}{\rho_{o}c} \sqrt{1-\cos\Delta\varphi_{BA}^{o}}$$

Thus, we see that when: $\Delta \varphi_{BA}^{o} = 0, \pm 2\pi, \pm 4\pi, \dots = \pm n_{even}\pi$ that: $\cos \Delta \varphi_{BA}^{o} = +1$ and thus:

$$\left| \tilde{p}_{tot} \left(x = 0, t \right) \right| = 2 \left| \tilde{A} \right|$$
 and: $\left| \tilde{u}_{tot}^{\parallel} \left(x = 0, t \right) \right| = 0$

i.e. we have complete constructive (destructive) interference associated with the two individual complex over-pressure (longitudinal particle velocity) amplitudes, respectively.

We also see that when: $\Delta \varphi_{BA}^o = \pm 1\pi, \pm 3\pi, \pm 5\pi, \dots = \pm n_{odd}\pi$ that: $\cos \Delta \varphi_{BA}^o = -1$ and thus: $\left| \tilde{p}_{tot} \left(x = 0, t \right) \right| = 0$ and: $\left| \tilde{u}_{tot}^{\parallel} \left(x = 0, t \right) \right| = 2 \left| \tilde{A} \right| / \rho_o c$

i.e. we have complete destructive (constructive) interference associated with the two individual complex over-pressure (longitudinal particle velocity) amplitudes, respectively.

Hence, we can also now see that when $|\tilde{R}| \equiv |\tilde{B}|/|\tilde{A}| \neq 1$, it is <u>*not*</u> possible to ever achieve <u>*complete*</u> constructive/destructive interference effects between the two individual right- and left-moving complex over-pressure and/or longitudinal particle velocity amplitudes.

Since:
$$\tilde{p}_{tot}(x,t) = \tilde{A} \left[1 + \left| \tilde{R} \right| e^{i\left(2kx + \Delta \varphi_{BA}^o\right)} \right] e^{i\left(\omega t - kx\right)}$$
 and: $\tilde{u}_{tot}^{\parallel}(x,t) = \frac{\tilde{A}}{\rho_o c} \left[1 - \left| \tilde{R} \right| e^{i\left(2kx + \Delta \varphi_{BA}^o\right)} \right] e^{i\left(\omega t - kx\right)}$

the phases of the complex total/resultant pressure and longitudinal particle velocity associated with the two counter-propagating 1-D monochromatic plane waves are given by:

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