and:

$$
\begin{split}\n\left|\tilde{a}^{\parallel}_{tot}\left(x,t\right)\right| &= \sqrt{\tilde{a}^{\parallel}_{tot}\left(x,t\right)} \cdot \tilde{a}^{\parallel*}_{tot}\left(x,t\right) \\
&= \frac{\left|\tilde{A}\right|}{\rho_{o}c} \sqrt{\left(1-\left|\tilde{R}\right|e^{i\left(2kx+\Delta\varphi_{BA}^o\right)}\right) \cdot \left(1-\left|\tilde{R}\right|e^{i\left(2kx+\Delta\varphi_{BA}^o\right)}\right)^{*}} \\
&= \frac{\left|\tilde{A}\right|}{\rho_{o}c} \sqrt{\left(1-\left|\tilde{R}\right|e^{i\left(2kx+\Delta\varphi_{BA}^o\right)}\right) \cdot \left(1-\left|\tilde{R}\right|e^{-i\left(2kx+\Delta\varphi_{BA}^o\right)}\right)} \\
&= \frac{\left|\tilde{A}\right|}{\rho_{o}c} \sqrt{1-\left|\tilde{R}\right|e^{i\left(2kx+\Delta\varphi_{BA}^o\right)} - \left|\tilde{R}\right|e^{-i\left(2kx+\Delta\varphi_{BA}^o\right)} + \left|\tilde{R}\right|^2} \\
&= \frac{\left|\tilde{A}\right|}{\rho_{o}c} \sqrt{1-\left|\tilde{R}\right| \left\{e^{i\left(2kx+\Delta\varphi_{BA}^o\right)} + e^{-i\left(2kx+\Delta\varphi_{BA}^o\right)}\right\} + \left|\tilde{R}\right|^2} \\
&= \frac{\left|\tilde{A}\right|}{\rho_{o}c} \sqrt{1-2\left|\tilde{R}\right|\cos\left(2kx+\Delta\varphi_{BA}^o\right) + \left|\tilde{R}\right|^2}\n\end{split}
$$

Thus, *e.g.* for an observer/listener's position $x = 0$, **and.** for *equal-strength* over-pressure amplitudes $|\tilde{A}| = |\tilde{B}| \Rightarrow |\tilde{R}| = |\tilde{B}|/|\tilde{A}| = 1$ (*i.e.* a *pure* standing wave!) these formulae simplify to:

$$
\left|\tilde{p}_{\text{tot}}\left(x=0,t\right)\right| = \sqrt{2}\left|\tilde{A}\right| \sqrt{1+\cos\Delta\varphi_{BA}^{\circ}} \text{ and: } \left|\tilde{a}_{\text{tot}}^{\parallel}\left(x=0,t\right)\right| = \frac{\sqrt{2}\left|\tilde{A}\right|}{\rho_{\text{o}}c} \sqrt{1-\cos\Delta\varphi_{BA}^{\circ}}
$$

Thus, we see that when: $\Delta \varphi_{BA}^{\circ} = 0, \pm 2\pi, \pm 4\pi, ... = \pm n_{even}\pi$ that: $\cos \Delta \varphi_{BA}^{\circ} = +1$ and thus:

$$
\left| \tilde{p}_{\text{tot}} \left(x = 0, t \right) \right| = 2 \left| \tilde{A} \right| \text{ and: } \left| \tilde{u}_{\text{tot}}^{\parallel} \left(x = 0, t \right) \right| = 0
$$

i.e. we have complete constructive (destructive) interference associated with the two individual complex over-pressure (longitudinal particle velocity) amplitudes, respectively.

We also see that when: $\Delta \varphi_{BA}^{\circ} = \pm 1\pi, \pm 3\pi, \pm 5\pi, ... = \pm n_{odd}\pi$ that: $\cos \Delta \varphi_{BA}^{\circ} = -1$ and thus: $|\tilde{p}_{\text{tot}}(x=0,t)|=0$ and: $|\tilde{u}_{\text{tot}}^{||}(x=0,t)|=2|\tilde{A}|/\rho_{\text{o}}c$

i.e. we have complete destructive (constructive) interference associated with the two individual complex over-pressure (longitudinal particle velocity) amplitudes, respectively.

Hence, we can also now see that when $|\tilde{R}| = |\tilde{B}|/|\tilde{A}| \neq 1$, it is **<u>not</u>** possible to ever achieve *complete* constructive/destructive interference effects between the two individual right- and leftmoving complex over-pressure and/or longitudinal particle velocity amplitudes.

Since:
$$
\tilde{p}_{tot}(x,t) = \tilde{A}\left[1 + |\tilde{R}|e^{i(2kx+\Delta\varphi_{BA}^o)}\right]e^{i(\omega t - kx)}
$$
 and: $\tilde{u}_{tot}^{\parallel}(x,t) = \frac{\tilde{A}}{\rho_{o}c}\left[1 - |\tilde{R}|e^{i(2kx+\Delta\varphi_{BA}^o)}\right]e^{i(\omega t - kx)}$,

the phases of the complex total/resultant pressure and longitudinal particle velocity associated with the two counter-propagating 1-D monochromatic plane waves are given by:

Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois 2002 - 2017. All rights reserved. $-9-$