

For “everyday” sound pressure levels  $SPL = L_p = 20 \log_{10} (p_{am}/p_o) \ll 134 \text{ dB}$ , corresponding to sound over-pressure amplitudes in “*free air*” at NTP of  $|\tilde{p}(\vec{r}, t)| \ll 100 \text{ RMS Pascals}$ , the **principle of linear superposition** holds, such that the total/resultant complex over-pressure and longitudinal particle velocity amplitudes respectively are:

$$\tilde{p}_{tot}(x, t) = \tilde{p}_A(x, t) + \tilde{p}_B(x, t) = \tilde{A}e^{i(\omega t - kx)} + \tilde{B}e^{i(\omega t + kx)} \text{ (Pascals)}$$

and:

$$\tilde{u}_{tot}^{\parallel}(x, t) = \tilde{u}_A^{\parallel}(x, t) + \tilde{u}_B^{\parallel}(x, t) = \tilde{u}_{A_o}^{\parallel}e^{i(\omega t - kx)} + \tilde{u}_{B_o}^{\parallel}e^{i(\omega t + kx)} = \frac{\tilde{A}}{\rho_o c}e^{i(\omega t - kx)} - \frac{\tilde{B}}{\rho_o c}e^{i(\omega t + kx)} \text{ (m/s)}$$

We can recast the above equations in terms of the dimensionless complex variable:

$$\tilde{R} \equiv \frac{\tilde{B}}{\tilde{A}} = \frac{|\tilde{B}|e^{i\varphi_B^o}}{|\tilde{A}|e^{i\varphi_A^o}} = \frac{|\tilde{B}|}{|\tilde{A}|}e^{i(\varphi_B^o - \varphi_A^o)} = |\tilde{R}|e^{i(\varphi_B^o - \varphi_A^o)} = |\tilde{R}|e^{i\Delta\varphi_{BA}^o} \text{ where: } \Delta\varphi_{BA}^o \equiv \varphi_B^o - \varphi_A^o$$

Thus:

$$\tilde{p}_{tot}(x, t) = \tilde{A} \left[ e^{i(\omega t - kx)} + |\tilde{R}|e^{i(\omega t + kx)} \cdot e^{i\Delta\varphi_{BA}^o} \right] = \tilde{A} \left[ 1 + |\tilde{R}|e^{i(2kx + \Delta\varphi_{BA}^o)} \right] e^{i(\omega t - kx)}$$

and:

$$\tilde{u}_{tot}^{\parallel}(x, t) = \frac{\tilde{A}}{\rho_o c} \left[ e^{i(\omega t - kx)} - |\tilde{R}|e^{i(\omega t + kx)} \cdot e^{i\Delta\varphi_{BA}^o} \right] = \frac{\tilde{A}}{\rho_o c} \left[ 1 - |\tilde{R}|e^{i(2kx + \Delta\varphi_{BA}^o)} \right] e^{i(\omega t - kx)}$$

We first calculate the **magnitudes** of the complex total/resultant over-pressure  $|\tilde{p}_{tot}(x, t)|$  and longitudinal particle velocity  $|\tilde{u}_{tot}^{\parallel}(x, t)|$ :

$$\begin{aligned} |\tilde{p}_{tot}(x, t)| &\equiv \sqrt{\tilde{p}_{tot}(x, t) \cdot \tilde{p}_{tot}^*(x, t)} \\ &= |\tilde{A}| \sqrt{\left(1 + |\tilde{R}|e^{i(2kx + \Delta\varphi_{BA}^o)}\right) \cdot \left(1 + |\tilde{R}|e^{i(2kx + \Delta\varphi_{BA}^o)}\right)^*} \\ &= |\tilde{A}| \sqrt{\left(1 + |\tilde{R}|e^{i(2kx + \Delta\varphi_{BA}^o)}\right) \cdot \left(1 + |\tilde{R}|e^{-i(2kx + \Delta\varphi_{BA}^o)}\right)} \\ &= |\tilde{A}| \sqrt{1 + |\tilde{R}|e^{i(2kx + \Delta\varphi_{BA}^o)} + |\tilde{R}|e^{-i(2kx + \Delta\varphi_{BA}^o)} + |\tilde{R}|^2} \\ &= |\tilde{A}| \sqrt{1 + |\tilde{R}| \left\{ e^{i(2kx + \Delta\varphi_{BA}^o)} + e^{-i(2kx + \Delta\varphi_{BA}^o)} \right\} + |\tilde{R}|^2} \\ &= |\tilde{A}| \sqrt{1 + 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) + |\tilde{R}|^2} \end{aligned}$$