Note further that:

$$\left\langle \tilde{I}_a^{\parallel}(x=0,t)\right\rangle_t = \frac{1}{2}p_o u_o^{\parallel} = \frac{1}{2}\frac{p_o^2}{\rho_o c} = \frac{1}{2}\rho_o c u_o^{\parallel 2} = \frac{1}{2}u_o^{\parallel 2} z_o \quad \left(Watts/m^2\right)$$

and again using the relation $p_o = \rho_o c u_o^{\parallel} = z_o u_o^{\parallel}$, that:

$$\tilde{I}_a^{\parallel}(x=0,\omega) = \left\langle \tilde{I}_a^{\parallel}(x=0,t)\right\rangle_t = c\left\langle w_{tot}^{inst}(x=0,t)\right\rangle_t = \frac{1}{2}p_o u_o^{\parallel} = \frac{1}{2}\frac{p_o^2}{\rho_o c} = \frac{1}{2}\rho_o c u_o^{\parallel 2} = \frac{1}{2}u_o^{\parallel 2} z_o \quad \left(Watts/m^2\right)$$

Example # 2: Two Counter-Propagating 1-D Plane Monochromatic Traveling Waves in "Free Air":

In this example, we imagine two ***un-equal*** strength harmonic (*i.e.* single-frequency) sound sources located at $x = \pm\infty$, with an observer/listener located near/at the origin x = 0. At the observer's location there will therefore be two 1-D monochromatic plane traveling waves propagating in opposite directions in "***free air***" (*i.e.* the Great Wide-Open).

The ***physical***, ***instantaneous time-domain*** over-pressure amplitudes associated with the right- and left-going 1-D monochromatic plane waves are individually ***purely real*** quantities:

$$p_A(x,t) = A\cos\left(\omega t - kx + \varphi_A^o\right) \text{ and } p_B(x,t) = B\cos\left(\omega t + kx + \varphi_B^o\right) \text{ with } A \neq B \text{ \{necessarily\}}$$

Note here that the frequency and position-independent phases φ_A^o and φ_B^o are explicitly included here to generalize the {relative} phase relation between the two counter-propagating 1-D monochromatic traveling waves, *e.g.* consider their phase relation at x = 0 and t = 0: $p_A(x=0,t=0) = A\cos\varphi_A^o$ and $p_B(x=0,t=0) = B\cos\varphi_B^o$.

The corresponding complex ***time-domain*** over-pressure amplitudes are:

$$\tilde{p}_A(x,t) = \tilde{A}e^{i(\omega t - kx)} \text{ and } \tilde{p}_B(x,t) = \tilde{B}e^{i(\omega t + kx)} \text{ with } \tilde{A} \neq \tilde{B} \text{ \{necessarily\}}$$

where $\tilde{A} = \left|\tilde{A}\right| e^{i\varphi_A^o} \equiv Ae^{i\varphi_A^o}$ and $\tilde{B} = \left|\tilde{B}\right| e^{i\varphi_B^o} \equiv Be^{i\varphi_B^o}$.

Each individual complex ***time-domain*** over-pressure amplitude satisfies its own Euler's equation:

$$\frac{\partial u_{A,B}^{\parallel}(x,t)}{\partial t} = -\frac{1}{\rho_o}\frac{\partial p_{A,B}(x,t)}{\partial x}$$

The corresponding right- and left-going complex ***time-domain*** longitudinal particle velocities are:

$$\tilde{u}_A^{\parallel}(x,t) = \frac{\tilde{A}}{\rho_o c}e^{i(\omega t - kx)} \equiv \tilde{u}_{A_o}^{\parallel} e^{i(\omega t - kx)} \text{ and: } \tilde{u}_B^{\parallel}(x,t) = -\frac{\tilde{B}}{\rho_o c}e^{i(\omega t + kx)} \equiv -\tilde{u}_{B_o}^{\parallel} e^{i(\omega t + kx)} \text{ (using } c = \omega/k\text{)}$$

Note the –*ve* sign in the left-going complex longitudinal particle velocity amplitude, which simple reflects the fact that it is propagating in the –*ve* x-direction.