The ***instantaneous*** potential, kinetic and total energy densities associated with a 1-D monochromatic traveling plane wave propagating in the +*x*-direction in "***free air***" at $x = 0$ are:

$$w_{potl}^{inst}\left(x=0,t\right) \equiv \frac{1}{2}\frac{1}{\rho_o c^2}\, p^2\left(x=0,t\right) = \frac{1}{2}\frac{1}{\rho_o c^2}\, p_o^2 \cos^2 \omega t = \frac{1}{\rho_o c^2}\, p_o^{rms\,2} \cos^2 \omega t$$

$$w_{kin}^{inst}\left(x=0,t\right) \equiv \frac{1}{2}\rho_o \vec{u}_{\|}\left(x=0,t\right)\bullet\vec{u}_{\|}\left(x=0,t\right) = \frac{1}{2}\rho_o u_o^{\|2} \cos^2 \omega t = \rho_o u_o^{\|rms\,2} \cos^2 \omega t$$

$$w_{tot}^{inst}\left(x=0,t\right) \equiv w_{potl}^{inst}\left(x=0,t\right) + w_{kin}^{inst}\left(x=0,t\right)$$

$$= \frac{1}{2}\frac{1}{\rho_o c^2}\, p_o^2 \cos^2 \omega t + \frac{1}{2}\rho_o u_o^{\|2} \cos^2 \omega t = \frac{1}{\rho_o c^2}\, p_o^{rms\,2} \cos^2 \omega t + \rho_o u_o^{\|rms\,2} \cos^2 \omega t$$

For this situation with a 1-D monochromatic traveling plane wave, we obtained the relation

$$z_a^{\|}\left(x\right) = \frac{p\left(x,t\right)}{u^{\|}\left(x,t\right)} = \frac{p_o}{u_o^{\|}} = \rho_o c \equiv z_o \quad \left(\Omega_a\right)$$

Thus we see again here that: $p_o = \rho_o c u_o^{\|} = z_o u_o^{\|}$. Using the <u>square</u> of this relation in the above instantaneous total energy density expression, we also see that {***<u>here</u>***}:

$$w_{tot}^{inst}\left(x=0,t\right) \equiv w_{potl}^{inst}\left(x=0,t\right) + w_{kin}^{inst}\left(x=0,t\right) = \frac{1}{\rho_o c^2}\, p_o^2 \cos^2 \omega t = \rho_o u_o^{\|2} \cos^2 \omega t$$

The ***time-averages*** of the ***instantaneous*** potential, kinetic and total energy densities associated with a 1-D monochromatic traveling plane wave propagating in the +*x*-direction in "***free air***" at $x = 0$ are:

$$\left\langle w_{potl}^{inst}\left(x=0,t\right)\right\rangle_t = \frac{1}{2}\frac{1}{\rho_o c^2}\, p_o^2 \underbrace{\left\langle \cos^2 \omega t\right\rangle_t}_{=1/2} = \frac{1}{4}\frac{1}{\rho_o c^2}\, p_o^2 = \frac{1}{2}\frac{1}{\rho_o c^2}\, p_o^{rms\,2} \quad \left(Joules/m^3\right)$$

$$\left\langle w_{kin}^{inst}\left(x=0,t\right)\right\rangle_t = \frac{1}{2}\rho_o u_o^{\|2} \underbrace{\left\langle \cos^2 \omega t\right\rangle_t}_{=1/2} = \frac{1}{4}\rho_o u_o^{\|2} = \frac{1}{2}\rho_o u_o^{\|rms\,2} \qquad \left(Joules/m^3\right)$$

$$\left\langle w_{tot}^{inst}\left(x=0,t\right)\right\rangle_t \equiv \left\langle w_{potl}^{inst}\left(x=0,t\right)\right\rangle_t + \left\langle w_{kin}^{inst}\left(x=0,t\right)\right\rangle_t = \frac{1}{4}\frac{p_o^2}{\rho_o c^2} + \frac{1}{4}\rho_o u_o^{\|2} = \frac{1}{2}\frac{p_o^{rms\,2}}{\rho_o c^2} + \frac{1}{2}\rho_o u_o^{\|rms\,2} \quad \left(Joules/m^3\right)$$

Again, using the square of the relation $p_o = \rho_o c u_o^{\|} = z_o u_o^{\|}$ in the above expression, we see that:

$$\left\langle w_{tot}^{inst}\left(x=0,t\right)\right\rangle_t \equiv \left\langle w_{potl}^{inst}\left(x=0,t\right)\right\rangle_t + \left\langle w_{kin}^{inst}\left(x=0,t\right)\right\rangle_t = \frac{1}{2}\frac{p_o^2}{\rho_o c^2} = \frac{1}{2}\rho_o u_o^{\|2} = \frac{1}{2c}\, p_o u_o^{\|} \quad \left(Joules/m^3\right)$$

Note that the ratio of the ***time-averaged*** potential energy density to the ***time-averaged*** kinetic energy density *e.g.* at $x = 0$ is equal to <u>unity</u> for a 1-D monochromatic traveling wave:

$$\frac{\left\langle w_{potl}^{inst}\left(x=0,t\right)\right\rangle_t}{\left\langle w_{kin}^{inst}\left(x=0,t\right)\right\rangle_t} = \frac{\frac{1}{4}\frac{p_o^2}{\rho_o c^2}}{\frac{1}{4}\rho_o u_o^{\|2}} = \frac{p_o^2}{\rho_o^2 c^2 u_o^{\|2}} = \frac{p_o^2}{z_o^2 u_o^{\|2}} = \frac{z_o^2}{z_o^2} = 1$$