

Note also that the purely real longitudinal **specific** acoustic impedance $z_a^{\parallel}(x) = \rho_o c \equiv z_o(\Omega_a)$ and/or the longitudinal **specific** acoustic admittance $y_a^{\parallel}(x) = 1/z_a^{\parallel}(x) = 1/\rho_o c \equiv y_o^{\parallel} = 1/z_o^{\parallel}(\Omega_a^{-1})$ and also the longitudinal velocity of energy flow, $\tilde{c}_a^{\parallel}(x) = c$ associated with a 1-D monochromatic plane wave propagating *e.g.* in the $+x$ -direction in “**free air**” have **no** spatial (*i.e.* x -) and/or frequency (*i.e.* f -) dependence.

The **instantaneous time-domain** longitudinal acoustic intensity associated with a 1-D monochromatic plane traveling wave propagating in the $+x$ -direction in “**free air**” is also a **purely real** quantity – *i.e.* plane wave acoustic energy is entirely in the form of **pure sound radiation** – no acoustic energy is {temporarily} stored “locally” at the point x . The **instantaneous time-domain** complex longitudinal acoustic intensity is:

$$I_a^{\parallel}(x, t) \equiv p(x, t) \cdot u^{\parallel}(x, t) = p_o u_o^{\parallel} \cos^2(\omega t - kx)$$

For an observer’s/listener’s position *e.g.* at $x = 0$:

$$I_a^{\parallel}(x = 0, t) = p(x = 0, t) u^{\parallel}(x = 0, t) = p_o u_o^{\parallel} \cos^2 \omega t$$

Noting that the time-averaged $\langle \cos^2 \omega t \rangle_t \equiv \frac{1}{\tau} \int_{t=0}^{t=\tau} \cos^2 \omega t dt = \frac{1}{2}$, the **time-averaged** **instantaneous time-domain** complex longitudinal sound intensity at the listener’s position $x = 0$ associated with a 1-D monochromatic plane traveling wave propagating in the $+x$ -direction in “**free air**” is:

$$\langle I_a^{\parallel}(x = 0, t) \rangle_t = p_o u_o^{\parallel} \langle \cos^2 \omega t \rangle_t = \frac{1}{2} p_o u_o^{\parallel}$$

We can also define **RMS amplitudes** of over-pressure and particle velocity in terms of their respective **peak amplitudes**: $p_o^{rms} \equiv \frac{1}{\sqrt{2}} p_o$ and $u_o^{rms} \equiv \frac{1}{\sqrt{2}} u_o^{\parallel}$. Thus, we see that the **RMS** value of the **instantaneous time-domain** longitudinal sound intensity at the listener’s position $x = 0$ associated with a 1-D monochromatic plane traveling wave propagating in the $+x$ -direction in “**free air**” is equal to the **time-averaged** longitudinal sound intensity at that point, *i.e.*:

$$I_a^{\parallel rms}(x = 0) = \langle I_a^{\parallel}(x = 0) \rangle_t = \frac{1}{2} p_o u_o^{\parallel} = p_o^{rms} u_o^{\parallel rms}$$

The reader can also easily verify for this example that the **frequency domain** active (*i.e.* real) and reactive (*i.e.* imaginary/quadrature) components of the **complex** longitudinal acoustic intensity associated with a 1-D monochromatic traveling plane wave propagating in the $+x$ -direction in “**free air**” are given by:

$$\tilde{I}_a^{\parallel}(x, \omega) \equiv \frac{1}{2} \tilde{p}(x, \omega) \tilde{u}^{\parallel*}(x, \omega) = \frac{1}{2} p_o \cancel{e^{i(\omega t - kx)}} u_o^{\parallel} \cancel{e^{-i(\omega t - kx)}} = \frac{1}{2} p_o u_o^{\parallel} = \frac{1}{2} p_o u_o^{\parallel} + 0i = \langle \tilde{I}_a^{\parallel}(x, t) \rangle_t$$

Here in **this** problem, note that: $\langle \tilde{I}_a^{\parallel}(x, t) \rangle_t = \langle \tilde{I}_{ar}^{\parallel}(x, t) \rangle_t + i \langle \tilde{I}_{ai}^{\parallel}(x, t) \rangle_t = p_o u_o^{\parallel} + 0i = p_o u_o^{\parallel}$ has **no** position (*i.e.* x -) dependence!