<u>Example # 1: 1-D Plane Monochromatic Traveling Wave Propagating in "Free Air"</u>:

In "***free air***", the ***instantaneous*** ***<u>time</u>-<u>domain</u>*** pressure at a space-time point (x,t) associated with a 1-D plane monochromatic traveling wave propagating *e.g.* in the +*x*-direction is a ***purely real*** quantity: $p(x,t)=p_o\cos(\omega t-kx)$.

The 1-D ***instantaneous*** ***<u>time</u>-<u>domain</u>*** longitudinal particle velocity (*i.e.* in the +*x*-/propagation direction) at the space-time point (x,t) associated with a 1-D plane monochromatic traveling wave is obtained via the {linearized} 1-D Euler equation for inviscid fluid flow:

$$\frac{\partial u^{\parallel}(x,t)}{\partial t}=-\frac{1}{\rho_o}\frac{\partial p(x,t)}{\partial x}=-\frac{p_o}{\rho_o}\frac{\partial\cos(\omega t-kx)}{\partial x}=-\frac{kp_o}{\rho_o}\sin(\omega t-kx)$$

Then:

$$u^{\parallel}(x,t)=-\frac{kp_o}{\rho_o}\int\sin(\omega t-kx)\,dt=+\frac{kp_o}{\omega\rho_o}\cos(\omega t-kx)=\frac{p_o}{\rho_o c}\cos(\omega t-kx)=u_o^{\parallel}\cos(\omega t-kx)$$

where we have used the relation $c=\omega/k=343\,m/s$ = speed of sound in {bone-dry} air @ NTP (obtained from the 1-D wave equation(s) for *p* or $u^{\parallel}$). Note also that: $u_o^{\parallel}=p_o/\rho_o c=p_o/z_o$.

Since $p(x,t)=p_o\cos(\omega t-kx)$ and $u^{\parallel}(x,t)=(p_o/\rho_o c)\cos(\omega t-kx)=u_o^{\parallel}\cos(\omega t-kx)$, we see that the ***instantaneous*** ***<u>time</u>-<u>domain</u>*** pressure and longitudinal particle velocity are ***<u>in-phase</u>*** with each other for a 1-D monochromatic plane wave propagating in "***free air***". This in turn implies that for ***harmonic*** (*i.e.* single-frequency) {*aka* monochromatic} plane waves, the longitudinal ***<u>specific</u>*** acoustic impedance, ***<u>specific</u>*** admittance and intensity will thus also be ***purely*** ***<u>real</u>*** quantities for a 1-D monochromatic plane wave propagating in "***free air***"

We then "complexify" the above ***instantaneous*** ***<u>time</u>-<u>domain</u>*** pressure and longitudinal particle velocity expressions to obtain their complex ***<u>time</u>-<u>domain</u>*** representations: $\tilde{p}(x,t)=p_o e^{i(\omega t-kx)}$ and $\tilde{u}^{\parallel}(x,t)=u_o^{\parallel}e^{i(\omega t-kx)}$. The longitudinal ***<u>specific</u>*** acoustic impedance associated with a 1-D monochromatic plane wave propagating *e.g.* in the +*x*-direction in "***free air***" is then easily seen to {also} be a ***purely*** ***<u>real</u>*** quantity:

$$\tilde{z}_a^{\parallel}(x)=\frac{\tilde{p}(x,t)}{\tilde{u}^{\parallel}(x,t)}=\frac{p_o\,\cancel{e^{i(\omega t-kx)}}}{u_o^{\parallel}\,\cancel{e^{i(\omega t-kx)}}}=\frac{p_o}{u_o^{\parallel}}=\frac{\cancel{p_o}}{\cancel{p_o}/\rho_o c}=\rho_o c\equiv z_o\ \ (\Omega_a)$$

Since {here} $\tilde{z}_a^{\parallel}(x)=\rho_o\tilde{c}_a^{\parallel}(x)$, we see that the longitudinal velocity of energy flow $\tilde{c}_a^{\parallel}(x)=c$ for a 1-D monochromatic plane wave propagating *e.g.* in the +*x*-direction in "***free air***".

Note that this acoustic sound field example is the electrical analog of a simple *AC* circuit, *e.g.* driven at constant voltage by a sine-wave generator with a ***purely real instantaneous*** *AC* voltage $V(t)=V_o\cos\omega t$ imposed across an ***ideal*** resistor of resistance $R\ (\Omega)$ (hence ***purely real*** impedance $\tilde{Z}=R+i0\,(\Omega_e)$) resulting in a ***purely real instantaneous*** *AC* current $I(t)=I_o\cos\omega t$ flowing through it.